

RADIATIVE PROPERTIES OF SUBSTANCES AT HIGH ENERGY DENSITY AND OPTIMIZING SOFT X-RAY SOURCES FOR ICF APPLICATIONS .

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Theoretical background

Fundamental theoretical study of substances at high energy density includes following items:

1. Gas dynamics.
2. Photon transport processes.
3. Equation of state.
4. The radiative opacity.

The items are usually joined in the framework of a gas dynamics code to study laser interaction with matter. As known, precise measurements of physical parameters are limited for laser-produced plasma. Therefore, gas dynamics codes, which contain the mentioned items, are used extensively to determine temperature profiles, density profiles and other plasma characteristics (depending on time) inside the laser target, within the target thickness [1,2] along with experimental equipment. The radiative opacity represents an important part of this study.

1. Orzechowski, T. J., Rosen, M. D., Korblum, M. D., Porter J. L., Suter, L. J., Thissen, A. R., Wallace, R. J. (1996). The Rosseland Mean Opacity of a Mixture of Gold and Gadolinium at High Temperatures. *Phys. Rev. Lett.* **77**, pp. 3545-3548.
2. Callachan-Miller, D.& Tabak, M. (2000). Progress in target physics and design for heavy ion fusion. *Physics of Plasmas.* **7**, No. 5, pp. 2083-2091.

The density – functional theory.

In the grand canonical ensemble, the equilibrium electron density provides minimum of the grand potential

$$\Omega = Sp\{\hat{W}(\hat{H} - \mu\hat{N} + \Theta \ln \hat{W})\},$$

where the density matrix has the form

$$\hat{W} = \exp\left[-\frac{\hat{H} - \mu\hat{N}}{\Theta}\right] / Sp\left[-\frac{\hat{H} - \mu\hat{N}}{\Theta}\right], \text{ here } \Theta \text{ is the}$$

plasma temperature.

$$|\Phi_A\rangle = |n_1^A, n_2^A, \dots, n_k^A, \dots\rangle, \quad \hat{H}_A = \sum_i \hat{T}_i + \sum_{i < j} v(\vec{r}_i, \vec{r}_j)$$

$$\hat{T}_i = -\frac{1}{2}\Delta_i - \frac{Z}{r_i} + V_{ext}^A(\vec{r}_i) \quad v(\vec{r}_i, \vec{r}_j) = \frac{1}{|\vec{r}_i - \vec{r}_j|}$$

$$\hat{H}_A = \sum_{m,n} \langle m | \hat{T} | n \rangle a_m^+ a_n + \frac{1}{2} \sum_{k_1, k_2, k_3, k_4} \langle k_1 k_2 | v | k_4 k_3 \rangle a_{k_1}^+ a_{k_2}^+ a_{k_3} a_{k_4}$$

$$\Omega = \sum_A W_A E_A - \mu \sum_A W_A N_A + \Theta \sum_A W_A \ln W_A$$

The general set of self-consistent field equations that describe the state of the whole ensemble of plasma atoms and ions.

$$\hat{T}_A \Psi_i(\vec{r}_1) + V_A(\vec{r}_1) \Psi_i(\vec{r}_1) - \sum_{j \leq K} n_{Aj}^c \int \Psi_j^*(\vec{r}_2) \frac{1}{|\vec{r}_1 - \vec{r}_2|} \Psi_i(\vec{r}_2) d\vec{r}_2 \Psi_j(\vec{r}_1) = \sum_j \lambda_{ij}^A \Psi_j(\vec{r}_1),$$

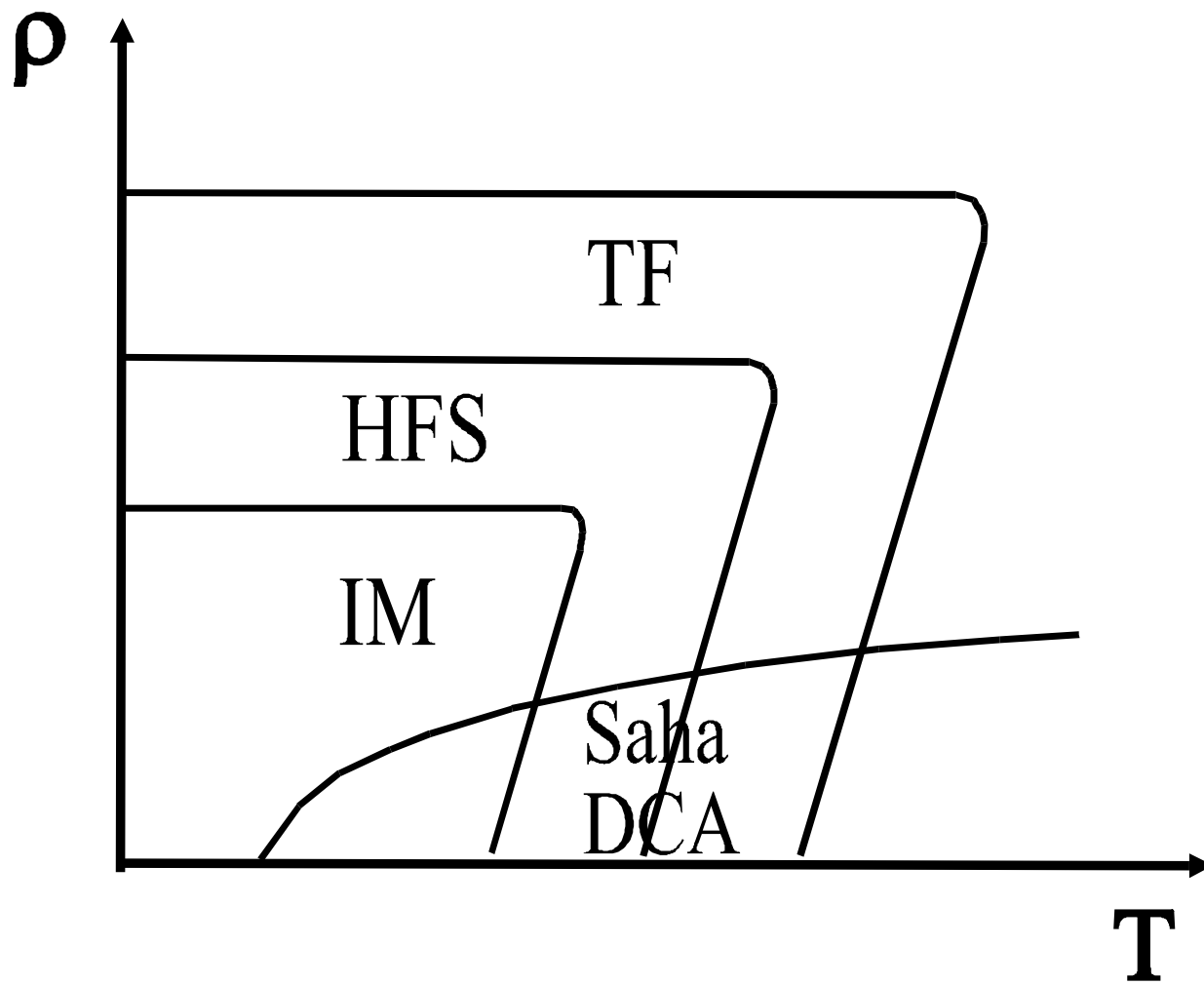
$$\bar{n}_A^f(\vec{r}, \vec{p}) = \left\{ \exp \left[\frac{1}{\Theta} \left(\frac{p^2}{2} - \frac{Z}{r} + V_A(\vec{r}) + V_{ext}^A(\vec{r}) - \mu \right) \right] + 1 \right\}^{-1}$$

- the equilibrium distribution for free electrons

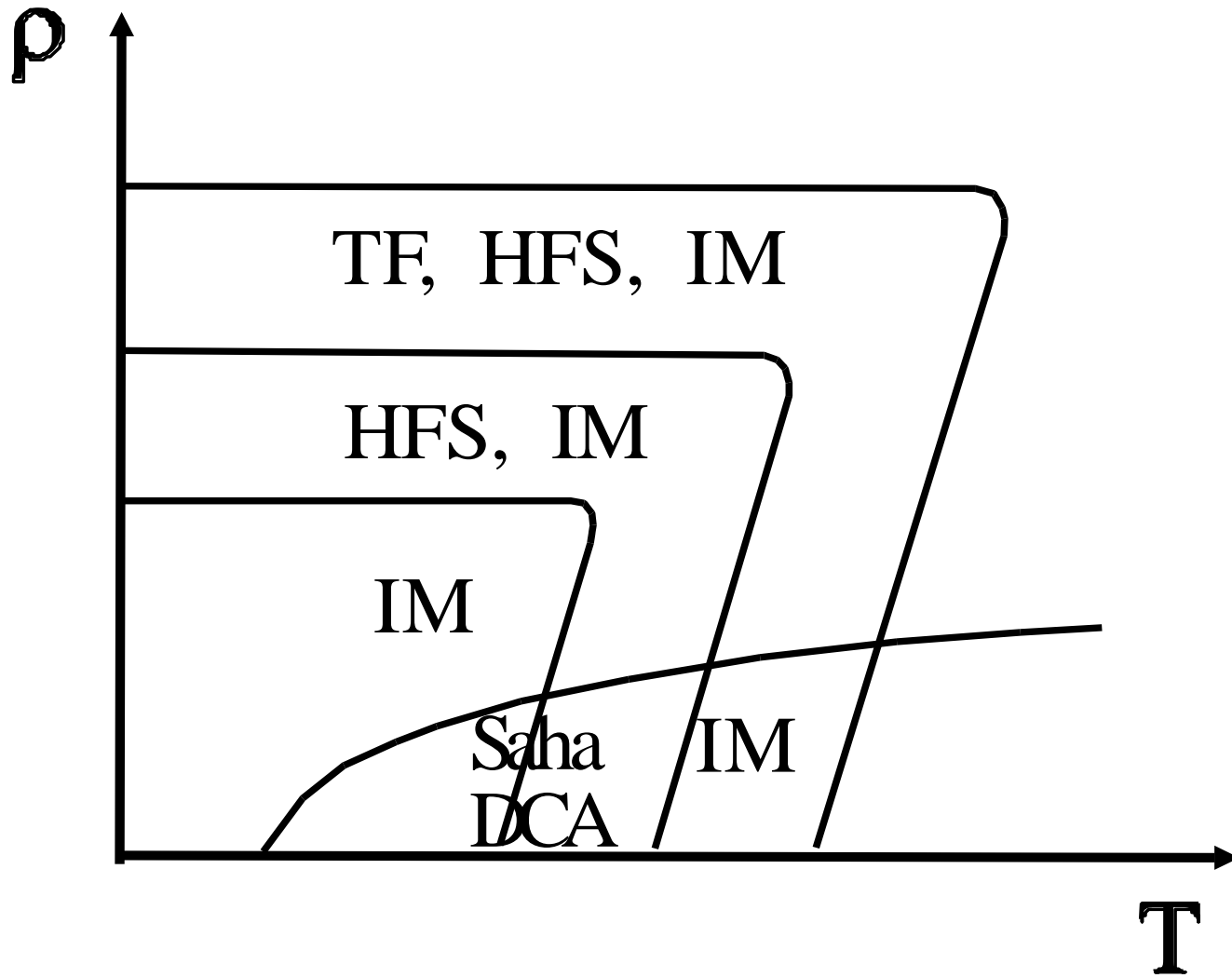
$$W_A = C g_A \exp \left\{ - \frac{E_A - \mu N_A}{\Theta} \right\},$$

$$\sum_A W_A N_A = Z.$$

Theoretical models of substances at high energy density.



Theoretical models of substances at high energy density.



Optically thick plasma.

Optically thick plasma can be produced by laser interaction with a thick solid target. The simple solution of the heat conductivity equation shows that radiation intensity $B(T)$ increases inversely proportional to the Rosseland mean free path, l_R (cm) :

$$B(T) \propto \frac{1}{l_R} . \quad (1)$$

A more precise approach (Orzechowski et al.) shows the hohlraum wall loss energy ΔE increases proportionally to the square root of the Rosseland mean free path.

$$\Delta E \propto [l_R]^{1/2} \quad (2)$$

The hohlraum wall efficiency increases with a reduction of this value. It can be achieved by decreasing the Rosseland mean free path.

Thus, an optimal chemical composition for optically thick plasmas can be achieved by minimizing the Rosseland mean free path.

Optically thin plasma.

Optically thin plasma can be produced by exploding wires. In this case, the outward energy flux increases inversely proportional to the Planck mean free path l_P .

$$j \propto \frac{1}{l_P} \quad (3)$$

The simple formula will be used for estimating relative radiation efficiency of different exploding wires made of two different materials A and B:

$$k = \frac{j^A}{j^B} = \frac{l_P^B}{l_P^A} \quad (4)$$

An optimal chemical composition for optically thin plasmas can be achieved by minimizing the Planck mean free path.

Results of calculations.

The spectral coefficient for x-ray absorption was calculated for plasma produced by laser interaction with gold hohlraum wall Au (black line). The coefficient for gold is relatively small in the energy interval $(3.5 < x < 8.5)$. The coefficient was also calculated for a composition, which is denoted as Composition 1 (red line). This composition was found using an optimization method. One can see, the interval is overlapped with spectral lines for Composition 1. It provides decreasing the Rosseland mean free path.

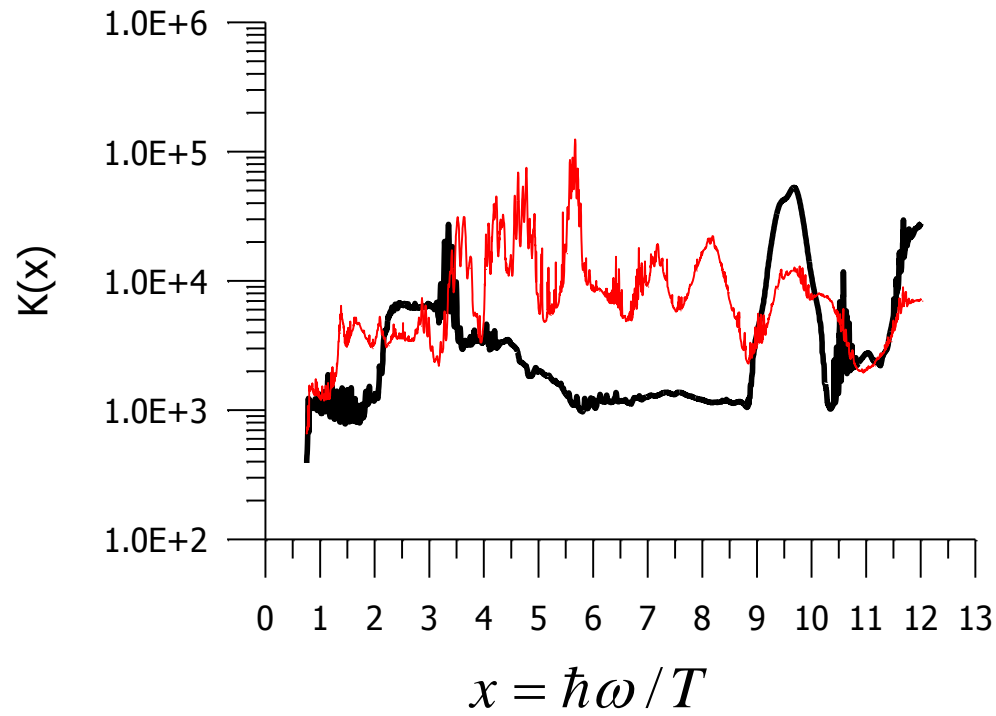


Fig.1 The spectral coefficient for X-rays absorption $K(x)$ (cm^2/g) calculated for Au (black line) and for Composition 1 (Au25.7%/W23.1%/Gd18.1%/Pr10.0%/Ba10.4%/Sb12.7%) at the temperature $T=250$ eV and the density $\tilde{\rho} = 1 \text{ g/cm}^3$.

Table 1. The hohlraum wall loss energy for different materials, compared to a Au hohlraum wall $\Delta E_{wall} / \Delta E_{Au}$ and to a AuGd hohlraum wall $\Delta E_{wall} / \Delta E_{AuGd}$.

Au		AuGd	
Material	$\frac{\Delta E_{wall}}{\Delta E_{Au}}$	Material	$\frac{\Delta E_{wall}}{\Delta E_{AuGd}}$
Au	1.00	Au/Gd(50:50)	1.00
Au:Gd	0.83	Au	1.25
U:At:W:Gd:La	0.65	Pb	1.28
U:Bi:W:Gd:La	0.65	W	1.25
U:Bi:Ta:Dy: Nd	0.63	Pb/Ta(70:30)	1.06
Th:Bi:Ta:Sm:Cs	0.68	Hg/Xe(50:50)	1.18
U:Pb:Ta:Dy:Nd	0.63	Pb/Ta/Cs(45:20:35)	1.01
U:Nb.14:Au:Ta:Dy	0.66	Pb/Hf/Xe(45:20:35)	1.00
Comp. 1	0.57	Comp. 1	0.75

Experimental and theoretical study of exploding wires.

The spectral coefficient for x-ray absorption was calculated for NiCr alloy (Ni80%/Cr20%) (black line) and for Alloy 188 (Cr21.72%/Ni22.92%/Fe2.24%/Co39%/W13.93%) (red line). One can see, the energy interval is overlapped with spectral lines of Alloy 188. It leads to decreasing the Planck mean, and Alloy 188 is more efficient material. Experimental study was carried out to test the theoretical results. Experimental measurements were made for the total energy yield B , and the experimental and theoretical coefficients of relative efficiency can be expressed in the form:

$$k^{\text{exp}} = B^{\text{Alloy188}} / B^{\text{NiCr}} \quad k^{\text{theor}} = j^{\text{Alloy188}} / j^{\text{NiCr}}$$

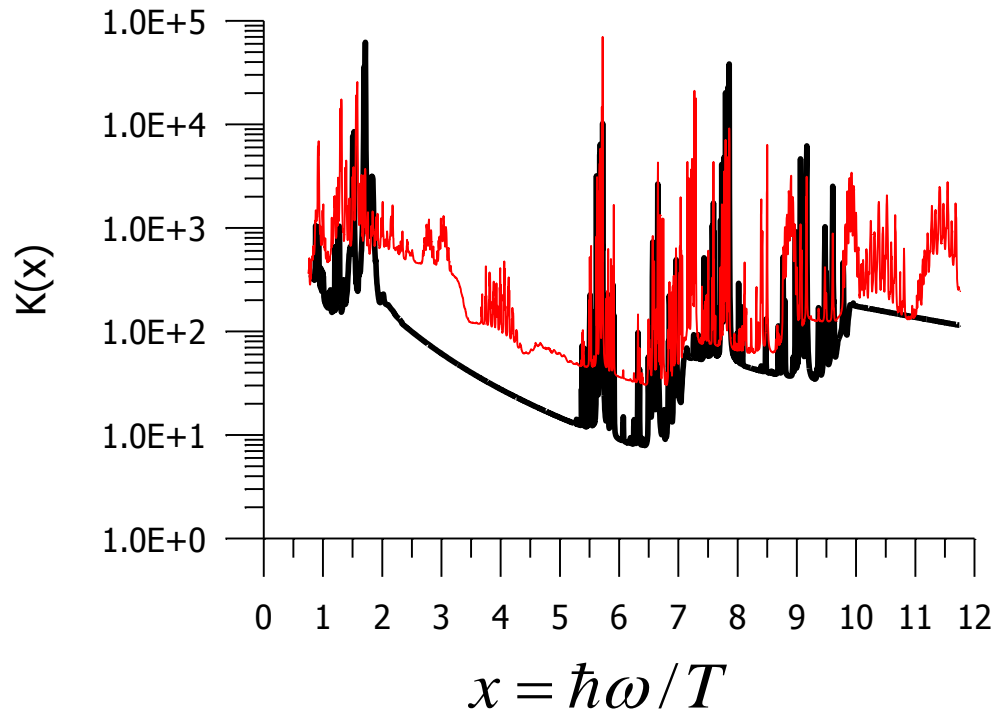


Fig. 2. The spectral coefficient for X-rays absorption $K(x)$ (cm^2/g) calculated for alloy (Ni80%/Cr20%) (black line) and for the composition Alloy 188 (Cr21.72%/Ni22.92%/Fe2.24%/Co39%/W13.93%) (red line) at the temperature $T=1$ keV and the density

$$\tilde{\rho} = \tilde{\rho}_{normal}.$$

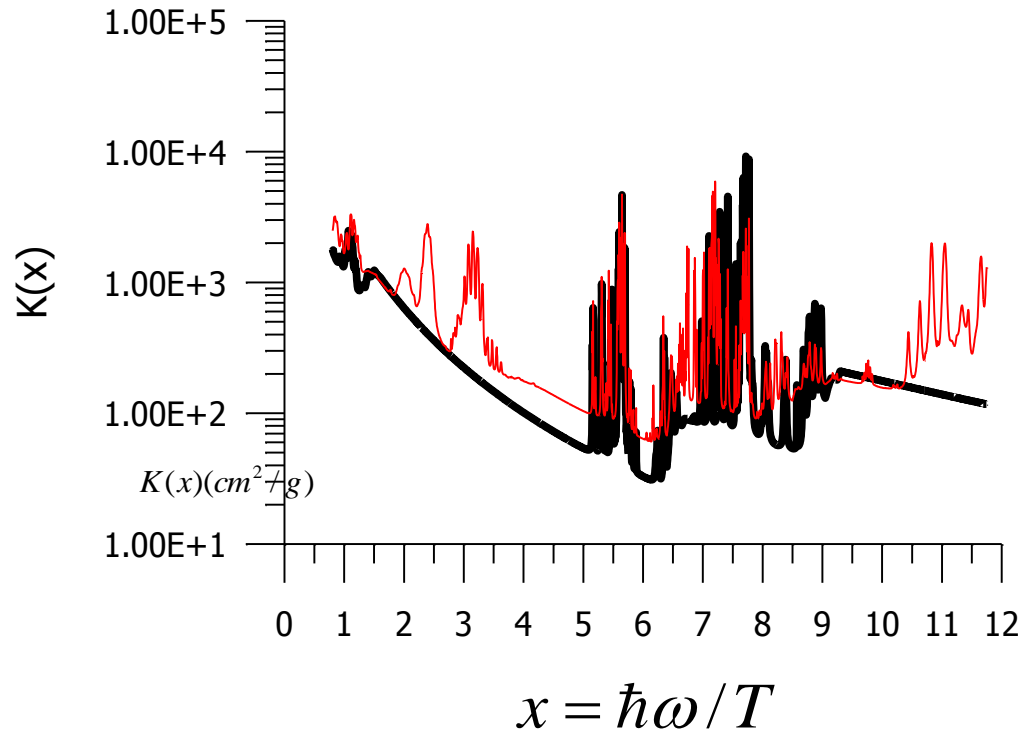


Fig. 3. The spectral coefficient of X-rays absorption calculated for NiCr (thick line) and for the composition Alloy 188 (Cr21.72%/Ni22.92%/Fe2.24%/Co39%/W13.93%) (thin line) at the temperature $T=1$ keV and the density

$$\tilde{\rho} = 10 * \tilde{\rho}_{normal}$$

Experimental and theoretical study of exploding wires.

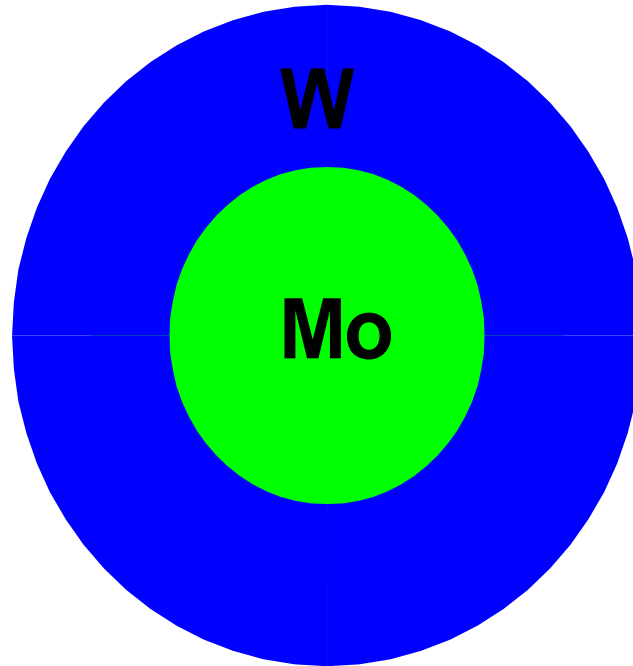
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$$k^{\text{exp}} = B^{\text{Alloy188}} / B^{\text{NiCr}} \quad . \quad k^{\text{theor}} = j^{\text{Alloy188}} / j^{\text{NiCr}}$$

Table 2. Theoretical and experimental results on the relative radiation energy yield from exploding wires made of alloy (Ni80%/Cr20%) and Alloy 188 (Cr21.72%/Ni22.92%/Fe2.24%/Co39%/ W13.93%).

Experiment	Theory			
	k^{exp}	Error bar	k^{theor}	$\Delta\%$
(E>1.5 keV)	1.736	20%	1.84	5.6%
(2.5<E<5 keV)	1.9	20%	1.84	3.2%

Structure of complex X- pinch plasma, where tungsten (W) and molybdenum (Mo) wires are used.



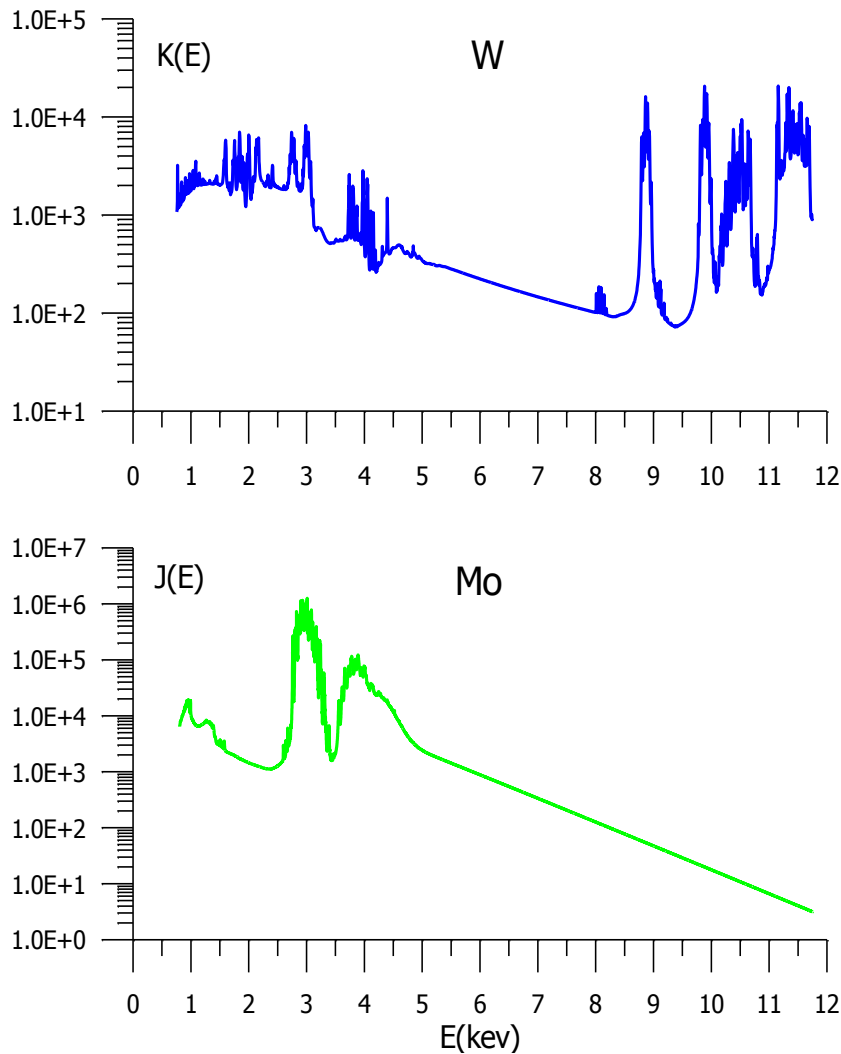


Fig. 4. The spectral coefficient for x-ray absorption $K(E)$ (cm^2/g) calculated for W (blue line) and the spectral coefficient for x-ray radiation $J(E)$ (a.u.) calculated for Mo (green line) at the plasma temperature $T=1$ keV and normal densities of W and Mo.

Complex X-pinch study.

Previous explanations were connected with big differences between the Rosseland or Planck mean free paths for different substances.

A more complicated case is connected with so-called complex X- pinch, where tungsten (W) and molybdenum (Mo) wires are used. Structure of complex X- pinch plasma gives more high radiation efficiency than pure tungsten. One of the possible theoretical explanations is connected with specific spectral coefficients for x-ray absorption and radiation.

The spectral coefficients for x-ray absorption and radiation were calculated for tungsten and molybdenum respectively. One can see, that energy interval of intense radiation for molybdenum coincides with the interval of intense absorption for tungsten. Therefore, practically all radiation of molybdenum is absorbed in tungsten. Experimental measurements confirmed that molybdenum spectral lines were not obtained outside the structure.

The effect leads to additional heating or preheating of tungsten plasma. One can note that 20% of additional temperature provides double increasing of radiation yield.

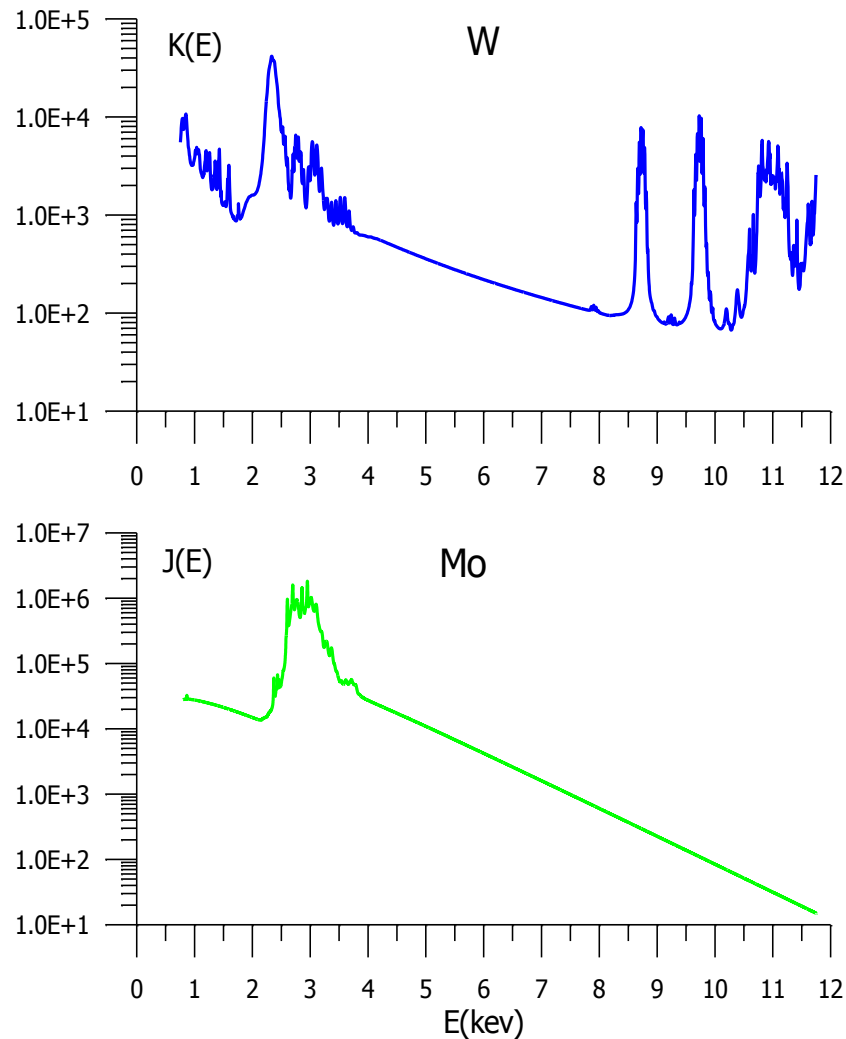


Fig 5. The spectral coefficient for x-ray absorption $K(E)$ (cm²/g) calculated for W (blue line) and the spectral coefficient for x-ray radiation $J(E)$ (a.u.) calculated for Mo (green line) for complex X-pinch at $T=1\text{keV}$ and the density $\tilde{\rho} = 10 * \tilde{\rho}_{norm}$

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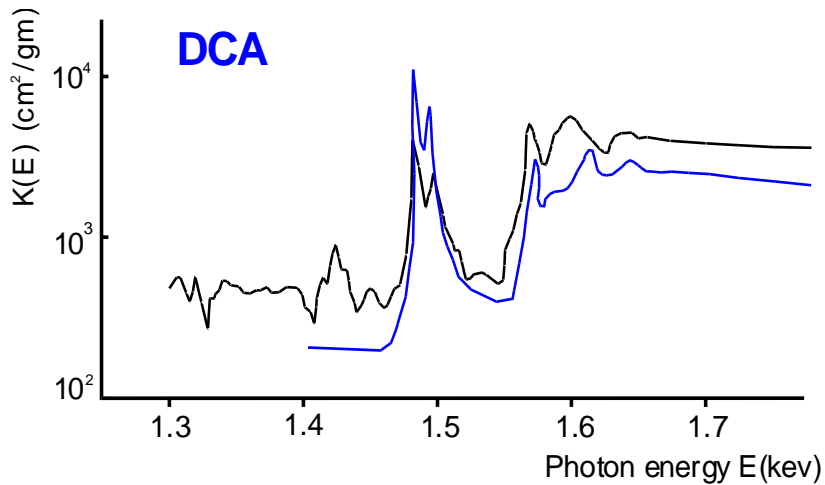
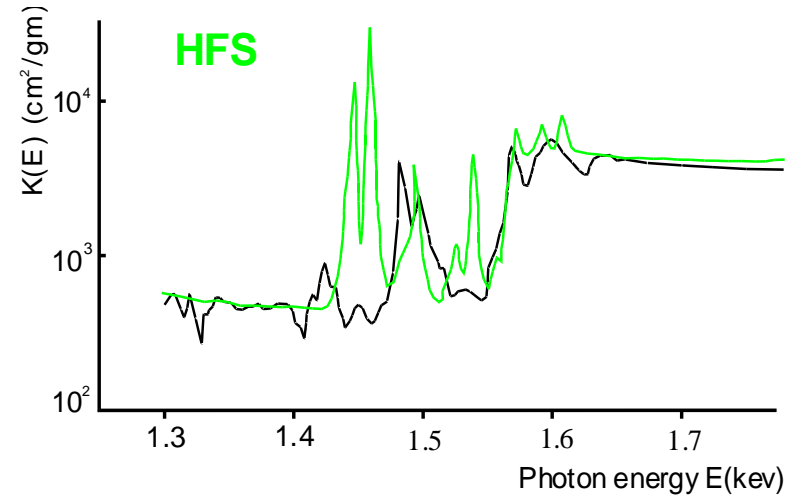
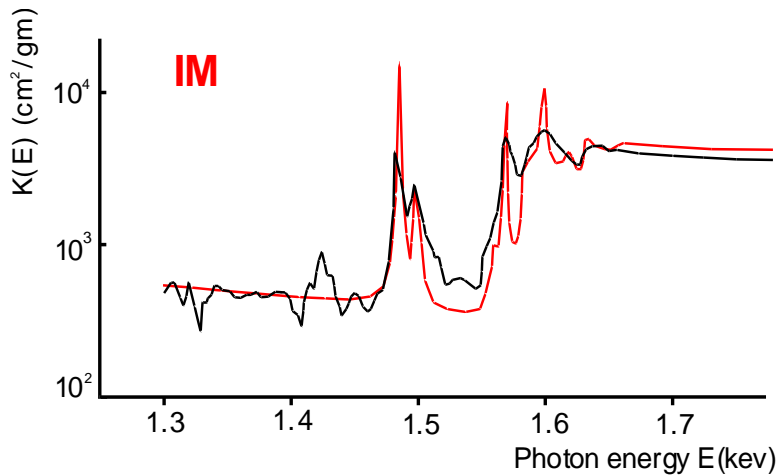
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CONCLUSIONS.

1. The Ion Model provides reliable quantum mechanical calculations of radiative opacity over a wide range of plasma temperature and density.
2. The optimization method, which has been developed to find out an effective complex material, really leads to better radiation efficiency of materials.
3. The Ion Model can be used to give theoretical explanations of experimental phenomena.

Appendix 1.



Comparison of experimental data (Davidson, S.J. et al.- black line) for the spectral coefficient of X-ray absorption and calculations (IM –red line, HFS-green line, DCA-blue line) in Al plasma at $T=18\text{eV}$, density 0.05 g/ccm .

Appendix 2.

The Planck mean free path (cm) calculated for NiCr and Alloy 188 at the temperature $T=1$ keV.

Density	NiCr	Alloy 188	k^{theor}
$0.1 d^{\text{nor}}$	$1.2 \cdot 10^{-2}$	$2.8 \cdot 10^{-3}$	4.06
d^{nor}	$2.9 \cdot 10^{-4}$	$2.2 \cdot 10^{-4}$	1.38
$10 d^{\text{nor}}$	$3.3 \cdot 10^{-5}$	$1.8 \cdot 10^{-5}$	1.84

$k^{\text{theor}} = j^{\text{Alloy188}} / j^{\text{NiCr}}$, d^{nor} is the normal density of material

Appendix 3.

The Rosseland mean free path (cm) calculated for different materials at the density 1 (g/ccm).

T (eV)	Au	Au50/ Gd50	Comp. 1.
200	$4.57 \cdot 10^{-4}$	$3.29 \cdot 10^{-4}$	$1.53 \cdot 10^{-4}$
250	$5.01 \cdot 10^{-4}$	$2.89 \cdot 10^{-4}$	$1.61 \cdot 10^{-4}$
300	$5.75 \cdot 10^{-4}$	$3.89 \cdot 10^{-4}$	$1.87 \cdot 10^{-4}$