Joint Institute for High Temperatures, Russian Academy of Sciences

Moscow Institute of Physics and Technology

NEW TECHNIQUE FOR ANALYSIS OF INTERPARTICLE INTERACTION IN NON-IDEAL DISSIPATIVE SYSTEMS

O.S. Vaulina, E.A. Lisin, A.V. Gavrikov, O.F. Petrov, V.E. Fortov

Outline

- Technique description
- Verifying the technique by conducting numerical simulations
- The first approbations for analysis of inter-grain interactions in plasma of rf- discharge

Present techniques for determining of inter-particle potentials in plasma

- Based on approximate integral equations for relationships between pair potential $U(l)$ and pair correlation function $g(l)$ (in statistical theory of liquids)

  O. S. Vaulina, O. F. Petrov, A. V. Gavrikov, V. E. Fortov, Plasma Physics Reports 33, 278 (2007).

  Features:

  1. *include a set of simplified assumptions of relationships between pair potential U(l) and pair correlation function g(l)*
  2. *complicate a correct analysis in strongly correlated systems*
  3. *small spatial range of identification of U(l) (correct analysis only in the neighborhood of the mean inter-particle distance)*

- Based on the measurements of a dynamic response of the dust system to various external perturbations

  V.E. Fortov, O.F. Petrov, A.D. Usachev, A.V. Zobnin, Phys. Rev. E **70,** 0046415 (2004)

  V.E. Fortov, A.V. Ivlev, S.A. Khrapak, A.G. Khrapak A, and G.E. Morfill, Phys. Reports **412,** 1 (2005).

  Features:

  1. *the necessity of prior information on the electric fields and the external forces*
  2. *sensible changes in the parameters of dust system*
  3. *measurements of interactions between two isolated particles only*

Advantages of the proposed technique

1. enables to determine a number of parameters of laboratory dusty-plasma systems at once :

   - a pair potential of inter-particle interaction
   - a friction coefficient due to collisions with surrounding gas neutrals
   - parameters of confining potential

2. the pair potential and the trap field are effectively recovered up to a distance of about half a typical size of dusty cloud (based on the processing of the numerical experiments)

3. is effective for strongly correlated systems (as` for liquid systems)

4. is a passive diagnostic technique which does not perturb an analyzed system

5. does not require any prior information on the relationship between $U(l)$ and $g(l)$ or on the form of electric fields and external forces

6. based only on the analysis of particle displacements that are easily fixed both in numerical and in real experiments

Procedure

1. Determination of displacements of particle coordinates $\vec{l}_{\mathrm{k}}(t)$

2. Calculation of particle velocities and accelerations

$$\vec{V}_{\mathrm{k}}(t_{\mathrm{m}}) = \frac{d\vec{l}_k}{dt} \cong \frac{\vec{l}_{\mathrm{k}}(t_{\mathrm{m}}) - \vec{l}_{\mathrm{k}}(t_{\mathrm{m-1}})}{\Delta t}$$

$$\vec{a}_{\mathrm{k}}(t_{\mathrm{m}}) = \frac{d^2\vec{l}_k}{dt^2} \cong \frac{\vec{V}_{\mathrm{k}}(t_{\mathrm{m}}) - \vec{V}_{\mathrm{k}}(t_{\mathrm{m-1}})}{\Delta t},$$

where, m is the number of video frame

3. Assignment of the force of pair interaction in the form of various combinations of power-law and exponential functions

$$F = \sum_{i=1}^{I_p} \{a_i l^{-(i+1)} + b_i l^{-i} \exp(\kappa l / l_p)\}$$

in particular:
- the inverse polinomial ($b_i = 0,\ I_p = 4$);
- the exponential nonlinear functions ($a_i = 0,\ I_p = 4$);
- the combined functions ($a_i \neq 0,\ b_i \neq 0,\ I_p = 4$)

4. The total force acting on a dust particle number k by the other particles

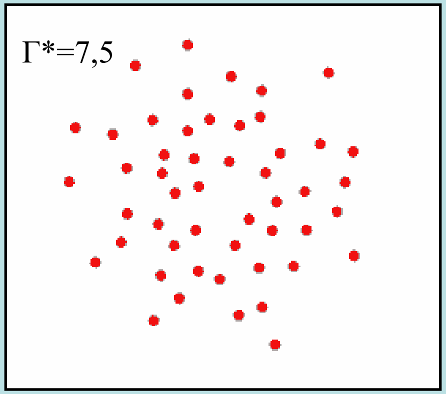
$$\vec{F}_{pp}{}^{k} = \sum_{j=1,\, j\neq k}^{N_d - 1} \sum_{i=1}^{I_p} \{a_i + b_i l_{kj} \exp(-\kappa l_{kj} / l_p)\} \frac{\vec{l}_k - \vec{l}_j}{l_{kj}^{i+2}}$$

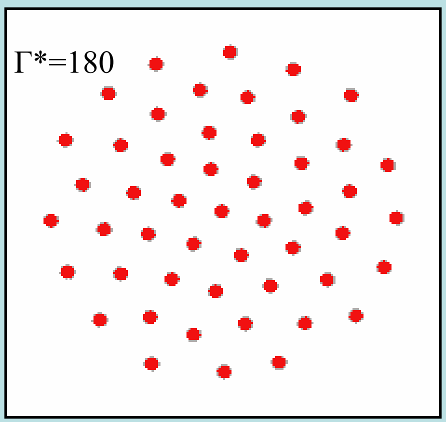
5. Approximation of the external confining force

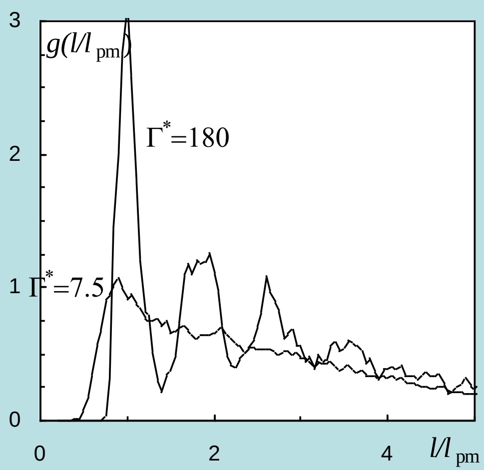
$$\vec{F}_{pt}{}^{k} = \sum_{i=1}^{I_t} b_i r^{i-1} \vec{r}_k$$

6. Search for the unknown coefficients $k,\ a_{\mathrm{i}},\ b_{\mathrm{i}},\ d_{\mathrm{i}}$ and ν_{fr} of the system of motion equations

$$M\vec{a}_k = -\nu_{fr} M\vec{V}_k + \vec{F}_{pp}{}^{k} + \vec{F}_{pt}{}^{k}$$







Numerical simulation

$$M\frac{d^2\vec{l}_k}{dt^2}=\sum_j F_{\text{int}}(l_{kj})\frac{\vec{l}_k-\vec{l}_j}{\vec{l}_{kj}}+\vec{F}_{ext}-M\nu_{fr}\frac{d\vec{l}_k}{dt}+\vec{F}_{ran}$$

ν_{fr} is the factor of particle friction, $l_{kj}=\left|\vec{l}_k-\vec{l}_j\right|$

$\vec{F}_{ran}$ is the Langevin force

$\vec{F}_{ext}=eZ\vec{E}(\vec{r})$ is the external electrical force of the trap, $\vec{E}=\alpha\vec{r}$

$F_{\text{int}}(l)=-\dfrac{\partial U}{\partial l}$ is the force of interparticle interaction

step $\Delta t=1/(20\max\{n_{\text{fr}},\omega^*\})$, where $\omega^*=(U''/\{l_{\text{p}}^3\pi M\})^{1/2}$

$$U = U_c\left[\text{c}_1\left(\frac{l_{\text{p}}}{l}\right)^{\text{n}_1}\exp\left(-k\frac{l}{l_{\text{p}}}\right)+\text{c}_2\left(\frac{l_{\text{p}}}{l}\right)^{\text{n}_2}+\text{c}_3\left(\frac{l_{\text{p}}}{l}\right)^{\text{n}_3}\right],\ \text{where } U_c = \frac{(eZ)^2}{l},$$

l_{p} is the mean interparticle distance, $k_{1(2)}$ is the shielding parameter

In particular

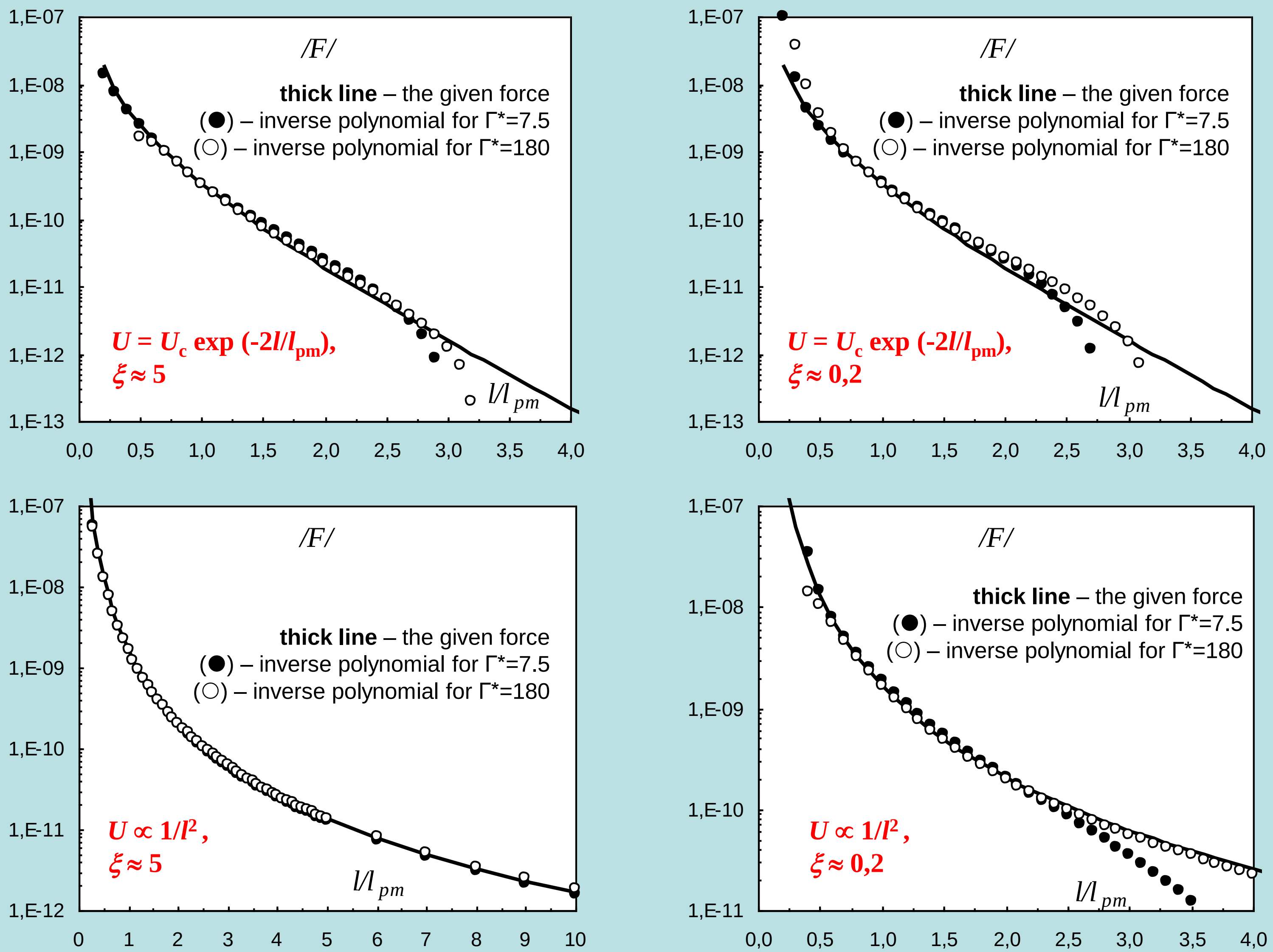
$$U_1=U_c\exp(-k\ l/l_{\text{p}}),\qquad k=1;\ 2;\ 4$$

$$U_2\propto\ 1/l^{\text{n}},\qquad \text{n}=2;\ 3$$

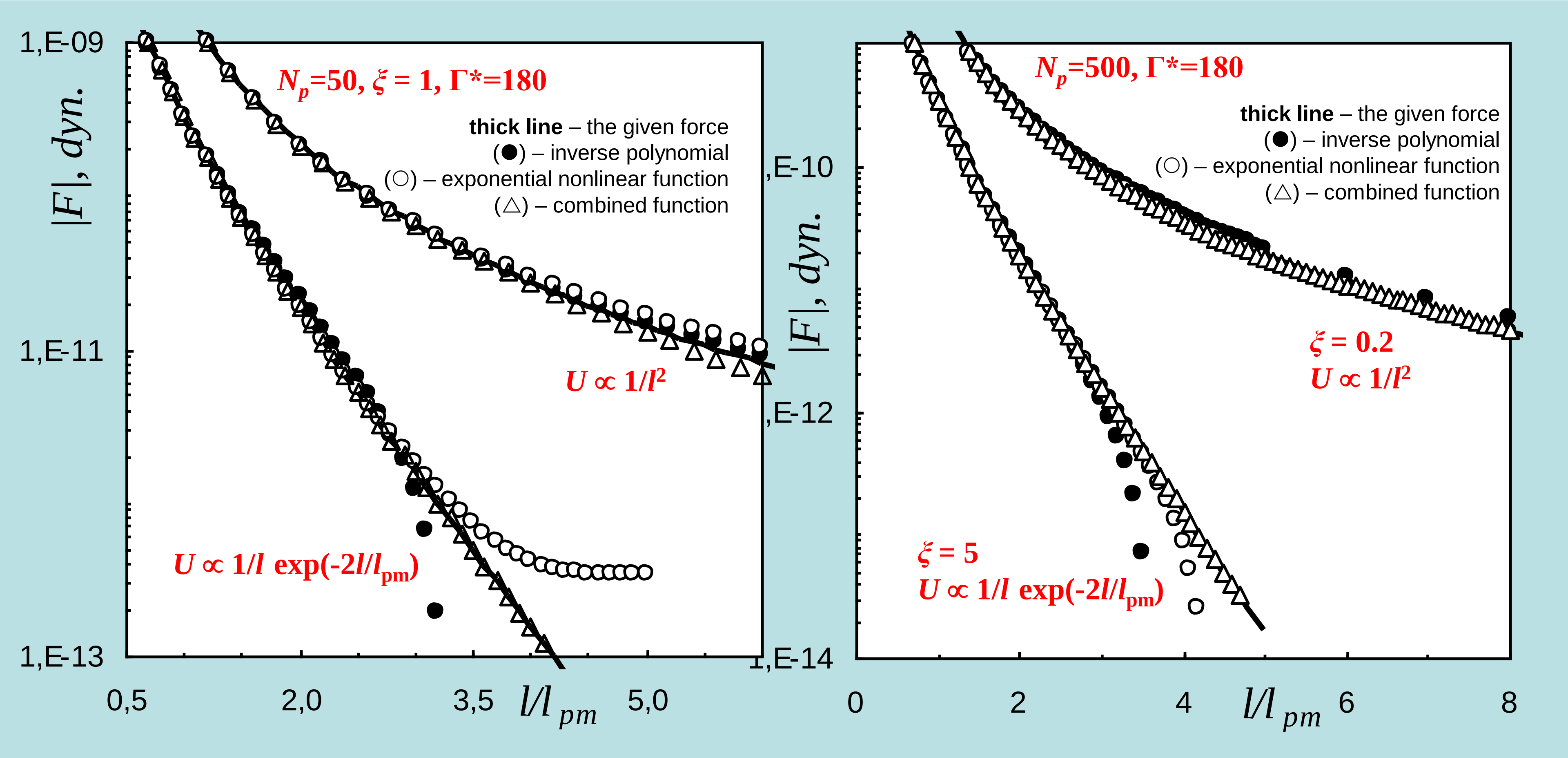
Varied parameters:

- the effective coupling par. $\Gamma^*=1.5\ l_p{}^2MU''(l_p)/(2T)$, $5\le\Gamma^*\le180$
- the scaling factor $\xi=\left|U''(l_p)\right|^{1/2}(\pi M)^{-1/2}\nu_{fr}{}^{-1}$, $0{,}2\le\xi\le5$
- the number of particles $2\le \text{N}_p\le500$

The results of recovery for 50 particles



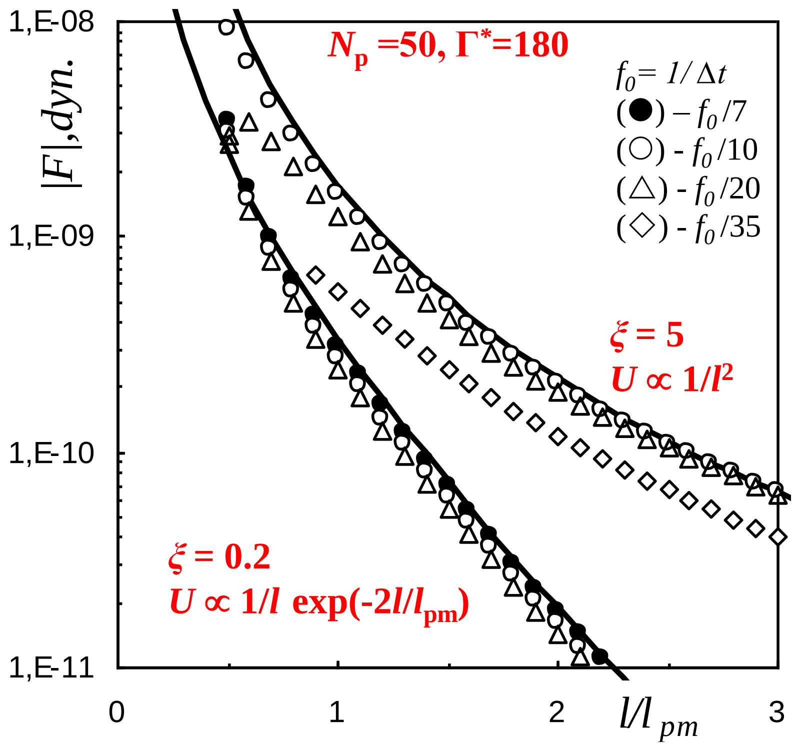
The results of recovery for 50 and 500 particles



Spatial range: $l_{\min} \le l \le l_{\max}$, $g(l_{\min}) \neq 0$, $F(l_{pm})/F(l_{\max}) \le 100\text{-}200$

Duration of experiment : the number of video frames $N_{vf} \geq 10\, N_{un}\, (l_{max}/\, l_{pm})^2\, /\min\{1;\xi\}$,
N_{un} is the number of unknown parameters.

Time and spatial resolutions



Time resolution:

$$f_{vr}^{\ min} = \max\{3\nu_{fr},\ 2\omega^*\}$$

Spatial resolution: $\delta l < (T/M)^{1/2} f_{vr}^{\ -1} \approx \omega^* l_{pm}\, f_{vr}^{\ -1}\, (3\pi/\{4\Gamma^*\})^{1/2}$

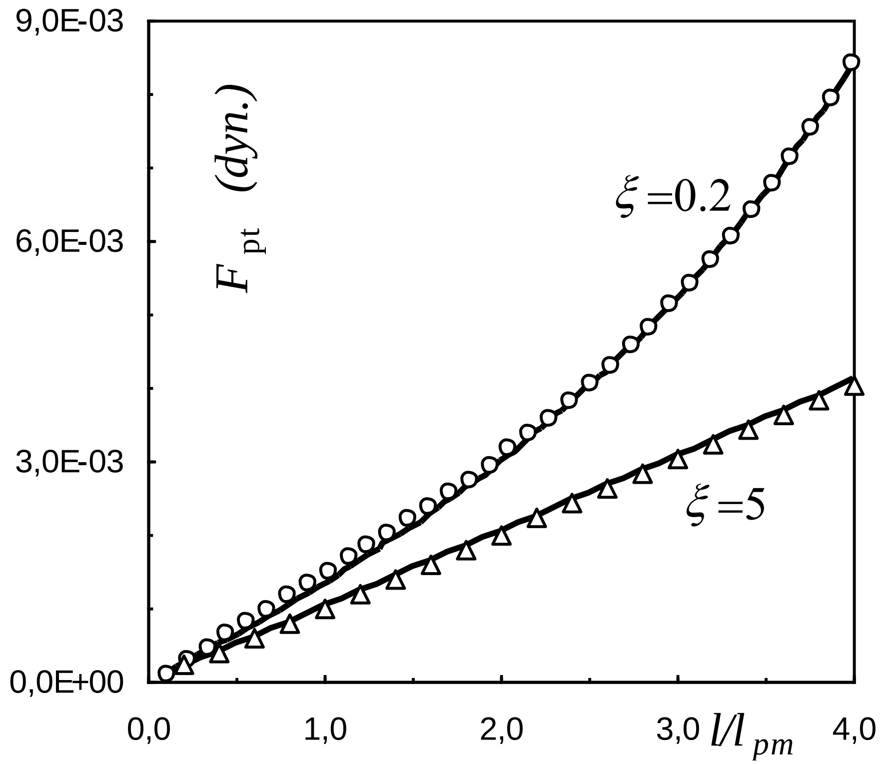
$$f_{vr}^{\ max} = \omega^*\, \delta_l^{\ -1} (3\pi/\{4\Gamma^*\})^{1/2}$$ for fixed $\delta_l = \delta l / l_{pm}$

Ex.: for $\xi \geq 1$ and $\delta_l = 0.005$

$[f_{vr}^{\ min};\ f_{vr}^{\ max}] = [2\omega^*;\ 112\omega^*]$ for $\Gamma^* = 7.5$,

$[f_{vr}^{\ min};\ f_{vr}^{\ max}] = [2\omega^*;\ 23\omega^*]$ for $\Gamma^* = 180$

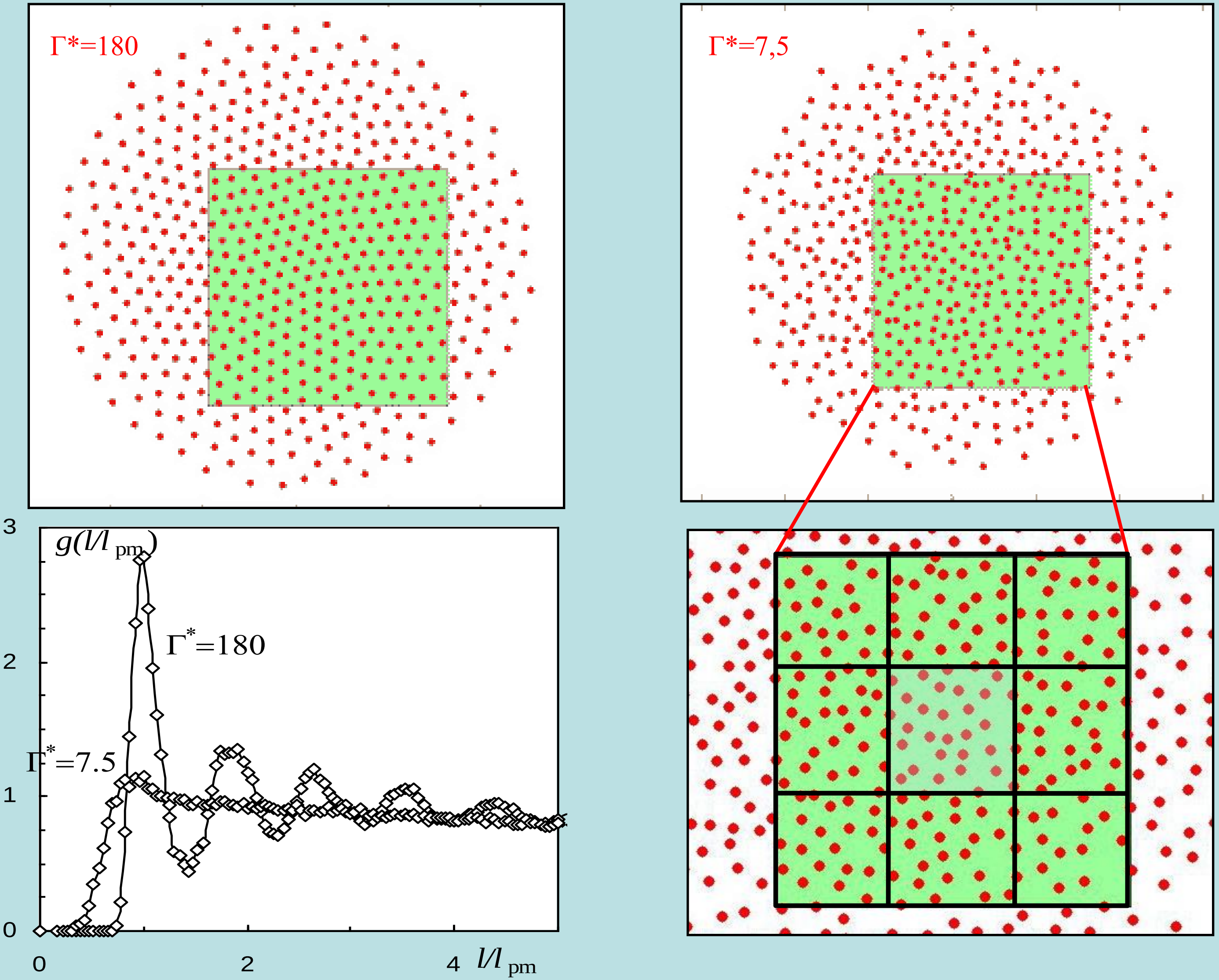
The recovery of external field



Confining force $F_{pt}(l/l_p)$ for 50 particles

Extended dust monolayer

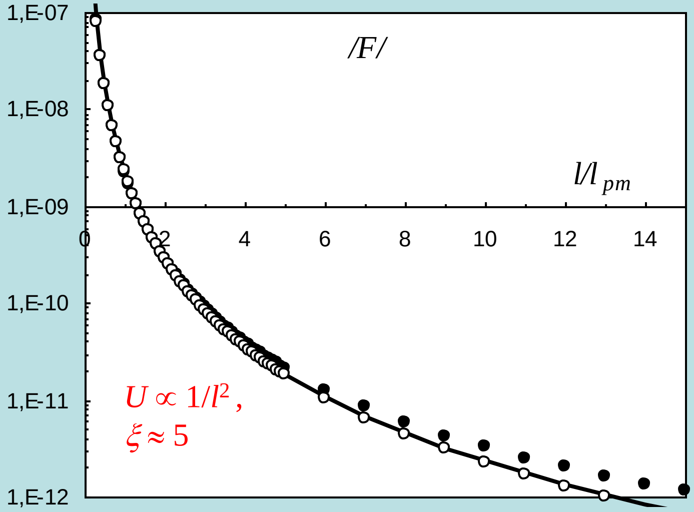
Modeling a situation when video camera registers only a part of a dust cloud:

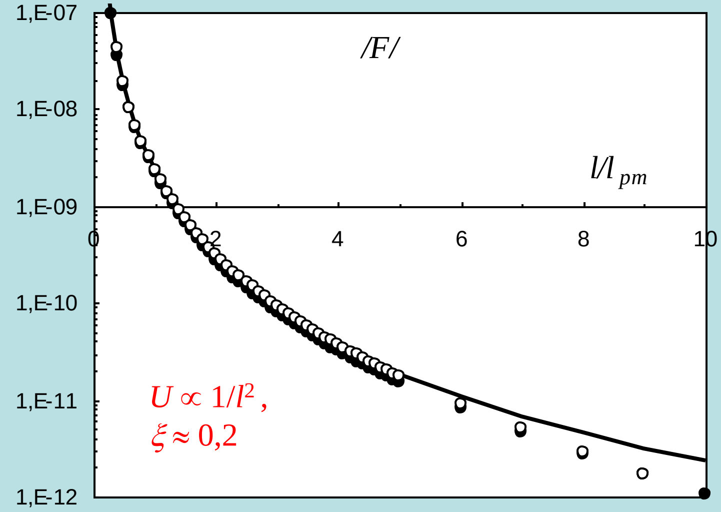


The scheme of analysis of a cloud fragment. Number of particles ~25% of the total number.

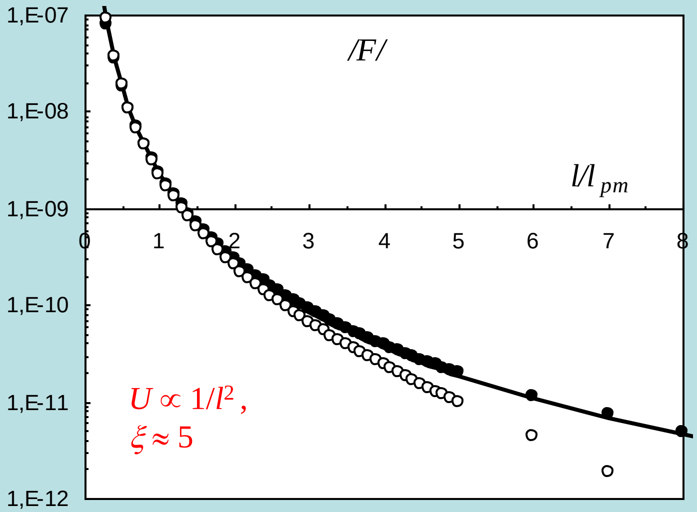
The results of recovery for 500 particles

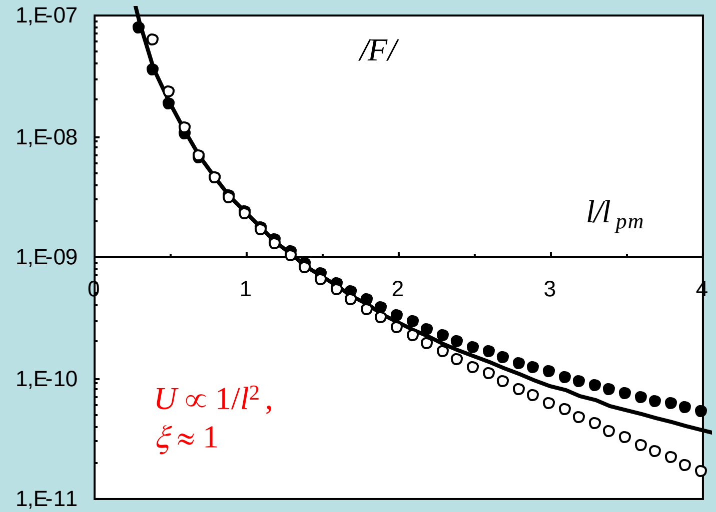
The results of analysis of all dusty structure





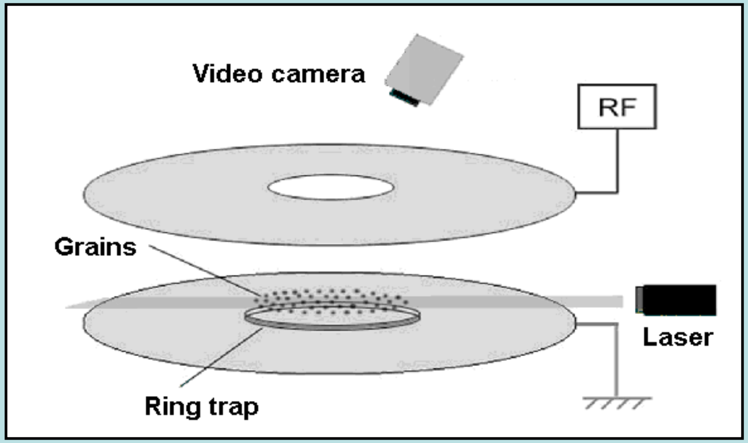
The results of analysis of the fragment





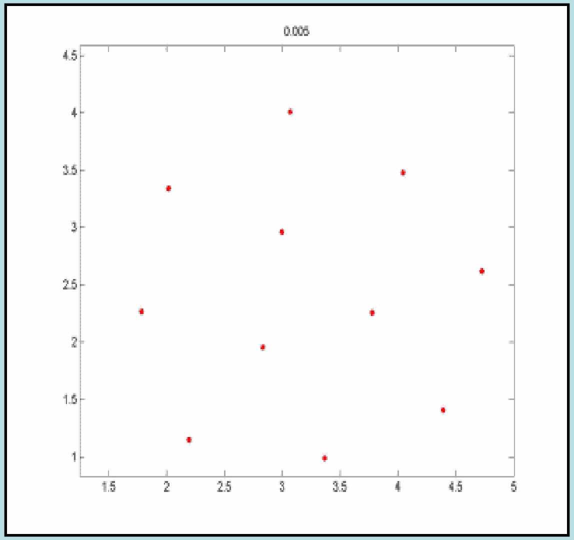
The force corresponding to the given potential is marked by a thick line; and also the recovered forces $\left|F(l/l_{pm})\right|$ for various parameters Γ^*: (●)- for Γ^*=7.5 , (○) – Γ^*=180

First experimental approbation

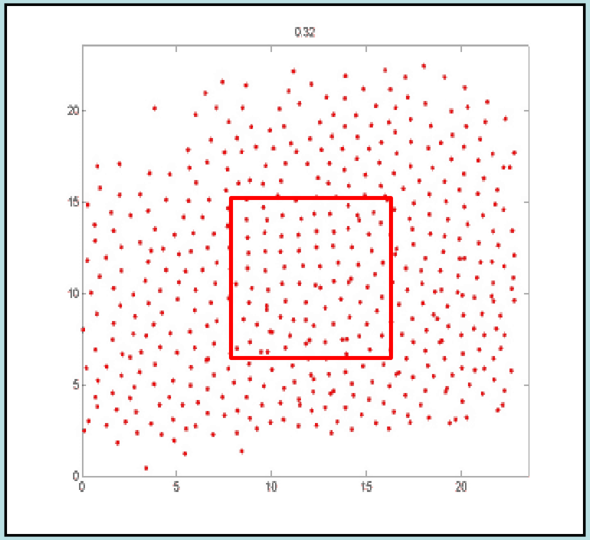


P = 0.01-0.06 Torr
a_p = 2.75 and 6.37 mkm
L_p ~ 750 - 1200 mkm

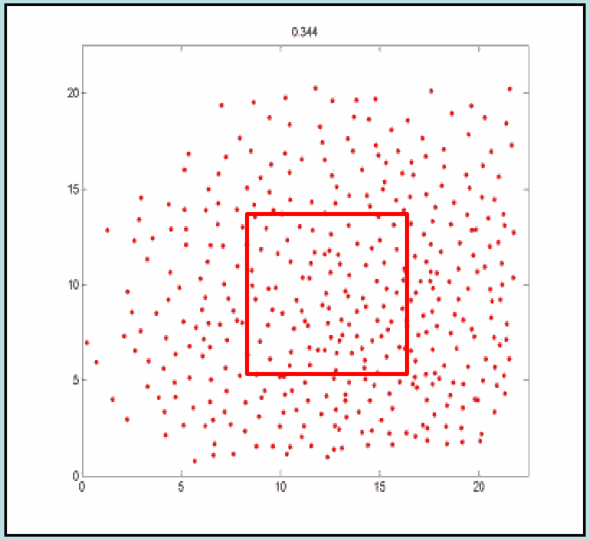
t_{exp} = 8-14 s
f_{vc} = 200-500 s^{-1}



1 – cluster (N_p=11, P=0.03 Torr)

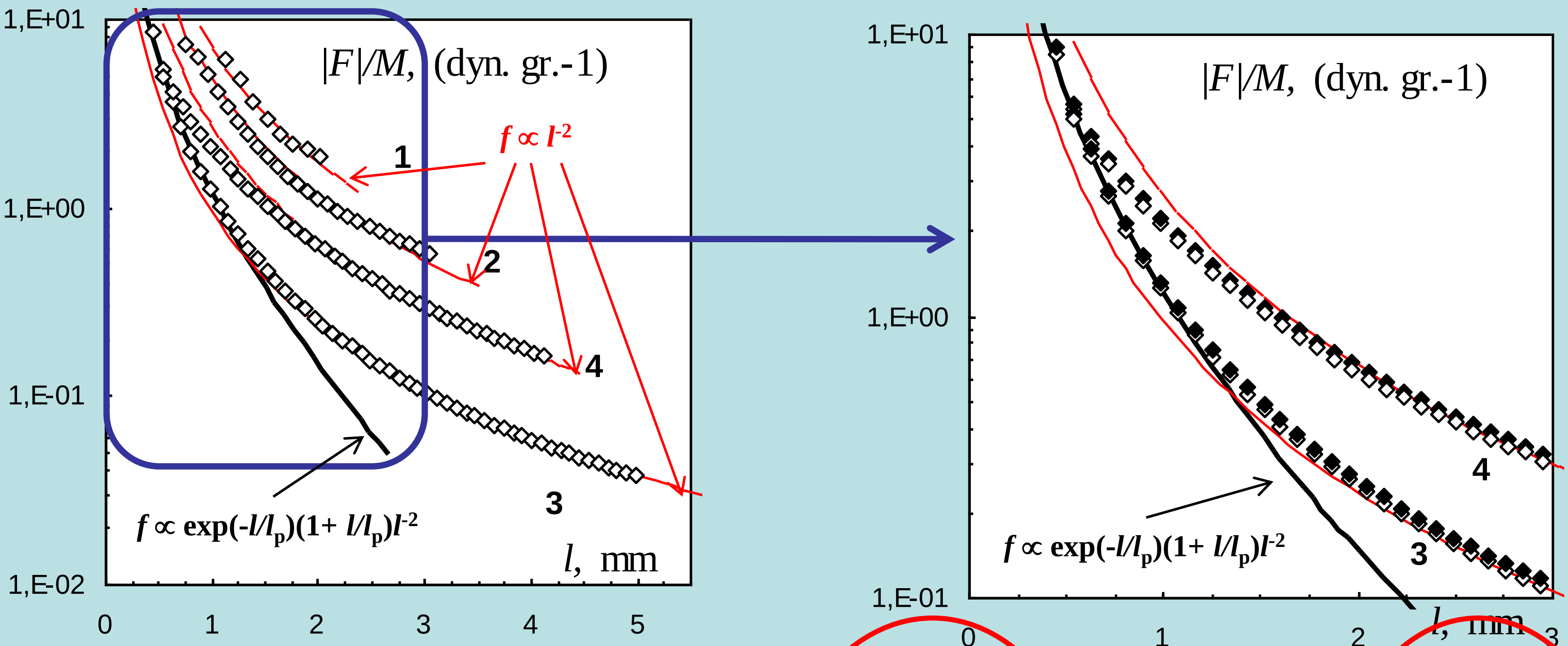


3 – part of dust monolayer (P=0.045 Torr)



4 – part of dust multilayer structure (P=0.022 Torr)

First experimental approbation



№	N_p	Γ^*	ν_{fr}, s^{-1}	ω_{c1}, s^{-1}	ω_{c2}, s^{-1}	$Z/1000$	α_1, $sm^{-1}s^{-2}$	α_2, $sm^{-1}s^{-2}$
1	11	~366	3.5±0.2	4.9±0.2	5.1±0.3	23.5±0.5	100	93
2	29	~420	8.1±0.6	5.9±0.2	6.1 ±0.3	18.6±0.2	94	99
3	~500(~2000)	~45	13 ± 1	3.4±0.15	3.6 ±0.3	2.98±0.2	7.2	5.8-6.6
4	~500(~2000)	~4.2	7.8±0.4	4.4±0.2	2.7 ±0.2	2.87±0.7	26.2	-

Analysis of the mass-transfer processes : O. S. Vaulina, X.G. Adamovich, O. F. Petrov, V. E. Fortov, Phys. Rev. E 77, 066404 (2008)

Summary

1. A new technique for determination of pair interaction forces is proposed

2. The technique doesn’t perturb an analyzed system, doesn’t require a prior information on the external fields or additional assumptions on the relationship between the pair correlation function and the pair interaction potential and can be applied to analyzing strongly correlated systems of interacting particles.

3. Numerical simulations in a wide range of the parameters typical for dusty plasma experiments were performed to verify the proposed procedure.

4. The first approbations of the proposed technique for analysis of inter-grain interactions in plasma of rf- discharge were performed

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Vaulina, Lisin, Plasma Phys. Rep., 33, 7 (2009)
Vaulina,.Lisin, Gavrikov, Petrov, Fortov, PRL 103, 035003 (2009)