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The kinetic model of laser plasma

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Dielectric function in the relaxation time approximation

- Boltzmann equation in linear approximation:

$$(-i\omega)\delta f + e\mathbf{E}\frac{\partial f_F(\mathbf{p})}{\partial \mathbf{p}} = -\nu_{ef}(p)\delta f$$

$$f_F(\mathbf{p}) = \frac{f_0(p)}{4\pi^3\hbar^3}$$

$$f_0(p) = \frac{1}{1+\exp\left(\frac{\varepsilon(p)-\mu}{T_e}\right)}$$

$$n_e = \frac{\sqrt{2}\left(m_e T_e\right)^{3/2}}{\pi^2\hbar^3} F_{1/2}\left(\frac{\mu}{T_e}\right)$$

$$F_{1/2}(\eta) = \int_0^\infty \frac{\sqrt{x}dx}{1+\exp(x-\eta)}$$

$$\varepsilon(\omega) = 1 + \frac{8\sqrt{2m_e}e^2}{3\pi\hbar^3\omega}\int_0^\infty \frac{\varepsilon^{3/2}d\varepsilon}{\omega + i\nu_{ef}(\varepsilon)}\frac{\partial f_0}{\partial \varepsilon}$$

Thermal conductivity and dc conductivity

- Boltzmann equation in linear approximation:

$$e\mathbf{E}\frac{\partial f_F(\mathbf{p},\mathbf{r})}{\partial \mathbf{p}}+\mathbf{v}\frac{\partial f_F(\mathbf{p},\mathbf{r})}{\partial \mathbf{r}}=-\nu_{ef}(p)\delta f$$

- Kinetic coefficients:

$$\mathbf{E}-\frac{1}{e}\nabla\mu=\frac{1}{\sigma}\mathbf{j}+\alpha\nabla T_e$$

$$\mathbf{q}'=\mathbf{q}-\left(\varphi+\frac{\mu}{e}\right)\mathbf{j}=\alpha T_e\mathbf{j}-\kappa\nabla T_e$$

$$\kappa=\frac{T_e^{5/2}}{3\pi^2\hbar^3}\sqrt{\frac{m_e}{2}}\left(I_{7/2}-\frac{I_{5/2}^2}{I_{3/2}}\right)$$

$$\sigma=\frac{e^2T_e^{3/2}}{3\pi^2\hbar^3}\sqrt{\frac{m_e}{2}}I_{3/2}$$

$$I_\beta(\eta)=\int_0^\infty\frac{x^\beta dx}{\nu_{ef}(x)}\mathrm{ch}^{-2}\left(\frac{x-\eta}{2}\right)$$

- Lorentz number:

$$L=\frac{\kappa}{\sigma T_e}$$

$$Le^2=\frac{I_{7/2}}{I_{3/2}}-\frac{I_{5/2}^2}{I_{3/2}^2}$$

Model of electron collision frequency

$$\nu_{ef}(\varepsilon) = \min\left\{ \sqrt{\frac{2\varepsilon}{m_e}} \frac{1}{r_0}, \frac{\varepsilon}{\pi\hbar}, \nu_C(\varepsilon), \nu_{met}(T_e, T_i, n_i) \right\}$$

- Coulomb collision frequency:

$$\nu_C(\varepsilon) = \sqrt{\frac{2}{m_e}} \frac{\pi e^4 Z^2 n_i \Lambda(\varepsilon)}{\varepsilon^{3/2}}$$

$$\Lambda(\varepsilon) = \max\left(2, \ln\frac{b_{\max}}{b_{\min}}\right)$$

$$b_{\max} = \max\left\{ r_0, \sqrt{\frac{T_{ef}}{m_e}} \frac{1}{\max\{\omega_p, \omega\}} \right\}$$

$$b_{\min}(\varepsilon) = \max\left\{ \frac{Ze^2}{2\varepsilon}, \frac{\hbar}{\sqrt{2m_e\varepsilon}} \right\}$$

$$T_{ef} = \sqrt{T_e^2 + T_{Fe}^2}$$

$$\frac{4\pi r_0^3 n_i}{3} = 1$$

Extreme case of degenerate electrons

$$\eta = \frac{\mu}{T_e} >> 1$$

- Fermi integral: $\int_0^\infty F(x) \frac{\partial f_0}{\partial x} dx = -F(\eta) - \frac{\pi^2}{6} F''(\eta) + O(\eta^{-4})$ $\quad f_0 = \frac{1}{1+\exp(x-\eta)}$

- Dielectric function: $\varepsilon = 1 - \frac{\omega_p^2}{\omega(\omega + i\nu_{met})}$

- DC conductivity: $\sigma = \frac{e^2 n_e}{m_e \nu_{met}}$

- Lorentz number: $L = \frac{\pi^2}{3e^2}$

Classical limit

$$\eta = \frac{\mu}{T_e} << -1$$

- Dielectric function:

$$\varepsilon(\omega) = 1 - \frac{16\sqrt{\pi}e^2 n_e}{3\omega m_e T_e^{5/2}} \int_0^\infty \frac{\varepsilon^{3/2} e^{-\varepsilon/T_e}}{\omega + i\nu_C(\varepsilon)} d\varepsilon$$

- DC conductivity:

$$\sigma = \frac{4\sqrt{2}T_e^{3/2}}{\pi^{3/2} Z e^2 \sqrt{m_e}\Lambda}$$

- Lorentz number:

$$L = \frac{4}{e^2}$$

Model of ionization

$$\frac{dn_e}{dt} = \sum_{i=0}^{Z_N - 1} \left[W_i\left(T_e, n_e\right) N_i - R_{i+1}\left(T_e, n_e\right) N_{i+1} \right]$$

$$n_e = Z n_i \qquad\qquad n_i = \sum_{i=0}^{Z_N} N_i$$

$$\frac{W_i\left(T_e, n_e\right) n_e}{R_{i+1}\left(T_e, n_e\right)} \square \frac{W_i\left(T_e, n_e^{eq}\right) n_e^{eq}}{R_{i+1}\left(T_e, n_e^{eq}\right)}$$

$$\frac{dn_e}{dt} = \sum_{i=0}^{Z_N - 1} W_i\left(T_e, n_e\right) N_i \left[1 - \frac{N_i^{eq}\left(T_e, n_i\right)}{N_{i+1}^{eq}\left(T_e, n_i\right)} \frac{N_{i+1}}{N_i} \frac{Z}{Z^{eq}} \right]$$

Ion distribution function

- Saha equation:

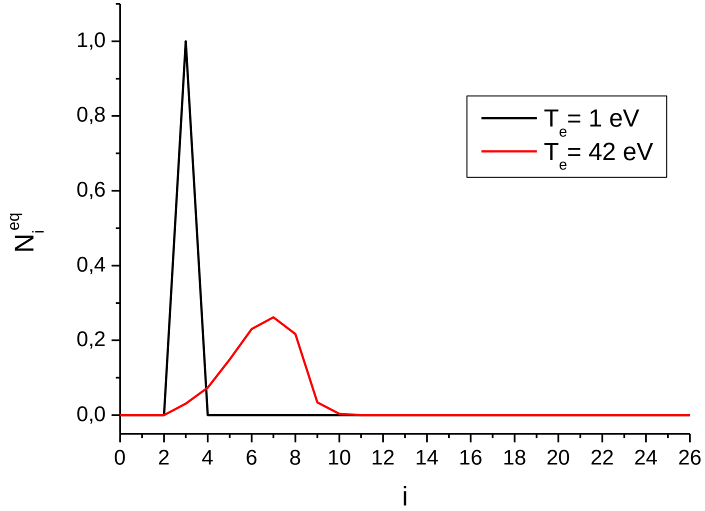
$$\frac{N^{eq}{}_{i}}{N^{eq}{}_{i+1}} = \frac{G_i}{G_{i+1}} \exp\left(\frac{I_i + \mu\left(Z^{eq}\right)}{T_e}\right)$$

$$I_i = I_{0i} - \Delta I_i$$

$$\Delta I_i = \frac{3Z_{ef}(i)e^2}{2k^2 r_0{}^3}\left[\left[\left(kr_0\right)^3 + 1\right]^{2/3} - 1\right]$$

$$k = \frac{\omega_p}{\sqrt{T_{ef}/m_e}}$$

- **M.S. Murillo, J.C. Weisheit // Physics Reports. 1998. V. 302. N 1. P. 1.**



Equilibrium distributions of ions in solid density iron at different electron temperatures

Ionization rate

$$\frac{N_i^{eq}}{N_{i+1}^{eq}}\frac{N_{i+1}}{N_i} \approx K_{eq}\left(T_e, Z\right) = \min\left\{1, \exp\left(\frac{I\left(Z\right) + \mu\left(Z^{eq}\right)}{T_e}\right)\right\}$$

$$I\left(Z\right) = \left(Z - \left[Z\right]\right) I_{[Z]+1} + \left(1 - Z + \left[Z\right]\right) I_{[Z]}$$

$$\frac{dZ}{dt} = \left(1 - \frac{Z}{Z^{eq}} K_{eq}\right) W$$

$$W = \frac{4.34}{\pi^2} q\left(Z\right) \int_1^{\infty} \frac{\ln\left(x\right) dx}{\left[1 + \exp\left(\frac{I\left(Z\right)x - \mu}{T_e}\right)\right]\left[1 + \exp\left(\frac{2\mu - I\left(Z\right)\left(x - 1\right)}{2T_e}\right)\right]^2}$$

Cluster heating and ionization

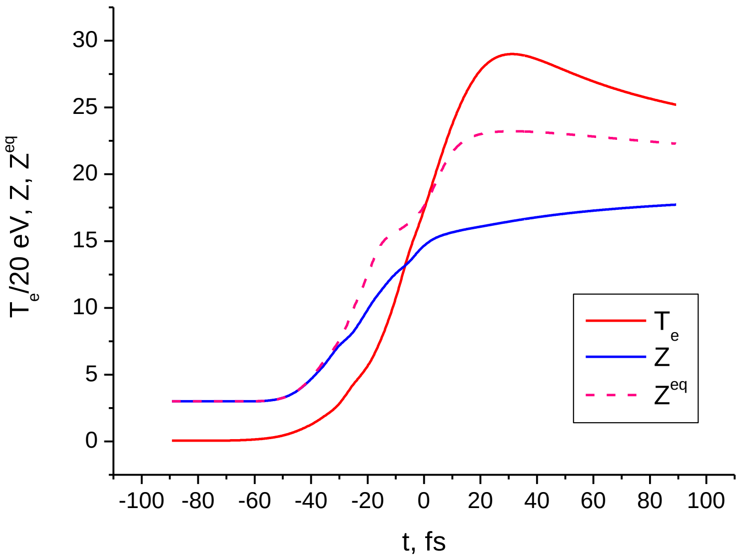
$$I(t)Q_a\left(T_e, Z\right) = \frac{dU_e}{dt} + \frac{dU_i}{dt},$$

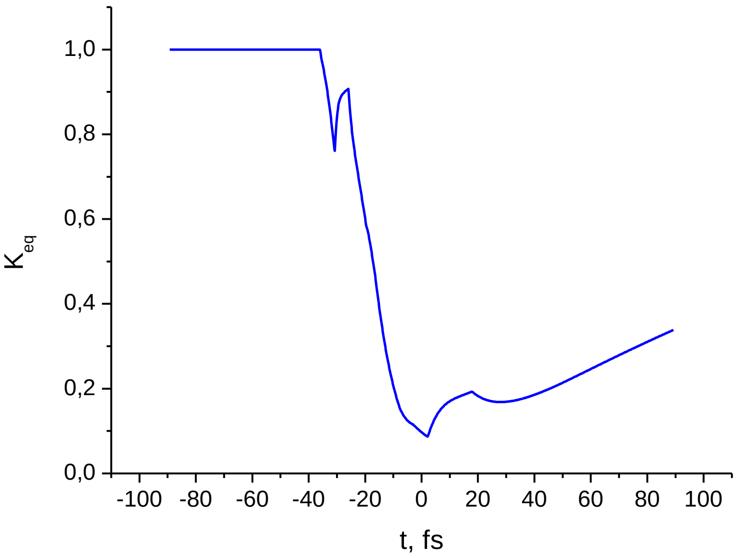
$$Q_a = -\frac{2\pi}{k_0^2}\sum_{n=1}^{\infty}(2n+1)\left[\mathrm{Re}\left(a_n^r\right) + \mathrm{Re}\left(b_n^r\right) + \left|a_n^r\right|^2 + \left|b_n^r\right|^2\right]$$

$$U_e = V\frac{\sqrt{2}}{\pi^2}T_e^{5/2}F_{3/2}\left(\frac{\mu}{T_e}\right)$$

$$\frac{dU_i}{dt} = VI\left(Z\right)\frac{dN_e}{dt}$$

Results of calculations





- Dynamics of T_e, Z and Z^{eq} at intensity 2×10^{17} W/cm^2 and iron cluster radius 25 nm (λ=0.8 μm, maximum of 40 fs laser pulse at t = 0)

Dynamics of the function $K_{eq}(T_e, Z)$

Conclusion

- Models of dielectric function, thermal conductivity and recombination, valid in the range of electron temperatures from degenerate plasma to classical one, are proposed

Thank you for attention!