Nonlinear Inverse Bremsstrahlung in Dense Plasmas

Alon Grinenko and Dirk O. Gericke

centre for fusion, space and astrophysics

Department of Physics,
University of Warwick, Coventry, United Kingdom

13th International Conference on Physics of Non-Ideal Plasmas
Chernogolovka, September 13 – 18, 2009

THE UNIVERSITY OF WARWICK

Motivation: Inverse bremsstrahlung in WDM

- Inverse Bremsstrahlung – Main laser absorption mechanism
- ICF target design – Symmetry conditions
- FI – Dense plasma created by the nanosecond pre-pulse
- Warm dense matter experiments and laser-cluster interactions

Collisional Absorption Calculation Approaches

- **The high-frequency limit: driver $\gg$ scattering frequency**
  - Silin ansatz[1] : Electrons freely oscillating in the external field
  - Mean field interactions – Vlasov-Poisson equations
  - Dynamic screening – Lindhard dielectric function
  - Dawson&Oberman[2]/Decker[3] absorption rate
- **The low-frequency limit: driver $\ll$ scattering frequency**
  - Electons bound to ions – their rest frames coincide
  - Binary scattering – Linearized Boltzmann approach
  - Static screening - Debye screening length
  - Generalized Drude conductivity formula
- **driver $\approx$ scattering frequency: none of these applies**

1 V.P. Silin, Sov. Phys. JETP 20, 1510 (1965).
2 J.M. Dawson, C. Oberman, Phys. Fluids 5, 517 (1962).
3 C. D. Decker, W. B. Mori, and J. M. Dawson, Phys. Plasmas **1**, 4043 (1994).

Collisional Absorption Calculation Approaches

- **The high-frequency limit: driver $\gg$ scattering frequency**
  - Silin ansatz[1] : Electrons freely oscillating in the external field
  - Mean field interactions – Vlasov-Poisson equations
  - Dynamic screening – Lindhard dielectric function
  - Dawson&Oberman[2]/Decker[3] absorption rate
- **The low-frequency limit: driver $\ll$ scattering frequency**
  - Electons bound to ions – their rest frames coincide
  - Binary scattering – Linearized Boltzmann approach
  - Static screening - Debye screening length
  - Generalized Drude conductivity formula
- **driver $\approx$ scattering frequency: none of these applies**

1 V.P. Silin, Sov. Phys. JETP 20, 1510 (1965).
2 J.M. Dawson, C. Oberman, Phys. Fluids 5, 517 (1962).
3 C. D. Decker, W. B. Mori, and J. M. Dawson, Phys. Plasmas **1**, 4043 (1994).

Collisional Absorption Calculation Approaches

- **The high-frequency limit: driver $\gg$ scattering frequency**
  - Silin ansatz[1] : Electrons freely oscillating in the external field
  - Mean field interactions – Vlasov-Poisson equations
  - Dynamic screening – Lindhard dielectric function
  - Dawson&Oberman[2]/Decker[3] absorption rate
- **The low-frequency limit: driver $\ll$ scattering frequency**
  - Electons bound to ions – their rest frames coincide
  - Binary scattering – Linearized Boltzmann approach
  - Static screening - Debye screening length
  - Generalized Drude conductivity formula
- **driver $\approx$ scattering frequency: none of these applies**

1 V.P. Silin, Sov. Phys. JETP 20, 1510 (1965).
2 J.M. Dawson, C. Oberman, Phys. Fluids 5, 517 (1962).
3 C. D. Decker, W. B. Mori, and J. M. Dawson, Phys. Plasmas **1**, 4043 (1994).

Equation of Motion of e- Fluid in an External Field

- ▷ Motivation: determine a frame in which e- distribution is ≈ equilibrium
- ▷ Kramers-Hennenberger frame – rest frame of e- fluid
- ▷ Equation of motion of the centre of mass of e- fluid in an external field

$$m\dot{\mathbf{V}} = -e\left[\mathbf{E}_{\text{ext}} + \langle \mathbf{E}(t) \rangle\right] - R(V)\mathbf{V}$$

- ◇ $\langle \mathbf{E}(t) \rangle$ – the induced **polarization** field
- ◇ $R(V)$ – the **e-i friction** coefficient

- ▷ **V** – the center of mass velocity of the e- fluid in the ion's rest frame

$$\mathbf{V}(t) \equiv \int \mathbf{v} F(\mathbf{r}, \mathbf{v}, t) d\mathbf{r} d\mathbf{v}$$

The Current Balance Equation

- Uniform, oscillating external field $\mathbf{E}_0 \sin(\omega_0 t)$
- The current balance equation

$$\frac{d\mathbf{j}}{dt} = \frac{\omega_p^2}{4\pi}\left[\mathbf{E_0}\sin\left(\omega_0 t\right) + \langle \mathbf{E}(t)\rangle\right] - \nu_{sc}(t)\mathbf{j}$$

- $\omega_p \equiv e^2 n_e / m$ – the plasma frequency
- $\mathbf{j} \equiv -e n_e \mathbf{V}$ – the current
- $\nu_{sc} \equiv R(V(t))/m$ – scattering frequency

- **Low-frequency** $\langle \mathbf{E}(t)\rangle = 0$, **linear** $\nu_{sc} = \mathrm{const}$ limit $\Longrightarrow$ **Drude**

$$\sigma = \frac{4\pi\omega_p^2}{\nu_{sc} - i\omega}$$

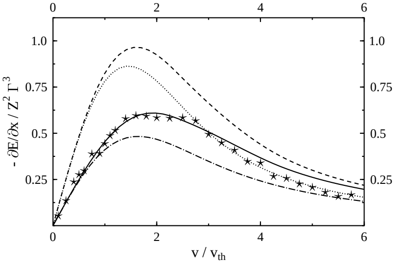
Solution of the Current Balance Equation

- In linear regime $\nu_{\text{sc}} = \text{const}$, the general solution is

$$\mathbf{j}(t) = \overbrace{\left[\mathfrak{Re}\{\sigma\}\mathbf{E}_0 \sin(\omega_0 t) - \mathfrak{Im}\{\sigma\}\mathbf{E}_0 \cos(\omega_0 t)\right]}^{\text{Free Current}} + \underbrace{\frac{\omega_p^2}{4\pi}\int_{-\infty}^{t} \langle \mathbf{E}(\tau)\rangle e^{\nu_{\text{sc}}(\tau - t)} d\tau}_{\text{Polarization Current}}$$

- Equivalent electric circuit:
  - Capacitor filled with dielectric material $\varepsilon_r > 1$
  - In parallel to non-zero resistivity

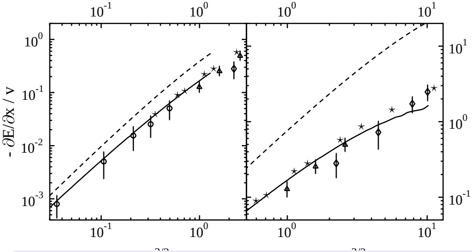
Stopping Power - the energy loss of beam per unit length



Stopping power: $-\partial E/\partial x$

- ▷ Ion beam $Z_b = 5$
- ▷ $n = 10^{20}\,\mathrm{cm}^{-3}$, $T = 1.6 \times 10^5$ K.
- ▷ RPA $--$; static Born $\cdots$;
- ▷ static T matrix $-\cdot-$;
- ▷ combined scheme —;
- ▷ PIC $\star$;

- ▷ For $v < v_{th}$ the stopping power $-\partial E/\partial x$ is linear in v
- ▷ $(-\partial E/\partial x)/v \equiv$ Friction Coefficient
- ▷ Self-similarity – $R(Z\Gamma^{3/2})$



$T_e = 10^6$ K (left) and $T_e = 5 \times 10^5$ K (right)

D. O. Gericke and M. Schlanges, PRE **60**,904 (1999)

Range of the Laser Field Amplitudes

- $R(\mathbf{V}) = \mathrm{const}$ can be assumed if: $V_{\mathrm{max}} < v_{\mathrm{th}}$
- Using the equation of motion

$$\frac{1 + (\nu_{\mathrm{sc}}/\omega_0)}{1 + (\nu_{\mathrm{sc}}/\omega_0)^2} \times v_0 < v_{\mathrm{th}}$$

where $v_{\mathrm{th}} = \sqrt{kT/m_e}$, and $v_0 = eE_0/m\omega_0$

High frequency $\nu_{\mathrm{sc}}/\omega_0 \ll 1$

- Free oscillating electrons
- Max. velocity $V_{\mathrm{max}} = v_0$
- Field amplitudes limited by v_{th}

Low frequency $\nu_{\mathrm{sc}}/\omega_0 \gg 1$

- Electrons are bound to the ions
- Max. velocity $V_{\mathrm{max}} = v_0/(\nu_{\mathrm{sc}}/\omega_0)$
- Larger laser field amplitudes

Absorption Rate

- Using the Poynting theorem, the **absorption rate** is (Silin)

$$\nu_{ei}(\omega_0) = \frac{4\pi\omega_0^2}{\omega_p^2} \frac{\langle \mathbf{j} \cdot \mathbf{E} \rangle}{\langle \mathbf{E} \cdot \mathbf{E} \rangle}$$

$$\Downarrow$$

$$\nu_{ei}(\omega_0) = \overbrace{\frac{\nu_{sc}}{1 + (\nu_{sc}/\omega_0)^2}}^{\text{Drude: Free Current Contribution}} + \underbrace{\frac{\omega_0^2}{1/2E_0^2} \left\langle \mathbf{E_0} \sin(\omega_0 t) \int_{-\infty}^{t} \langle \mathbf{E}(\tau) \rangle e^{\nu_{sc}(\tau - t)} d\tau \right\rangle}_{\text{DO/Decker } et\ al.\text{: Polarization Current Contribution}}$$

Calculation of the Polarization Field

- Polarization field is found from the solution of Vlasov–Poisson equations
- The electron distribution function F is determined by:

$$\frac{\partial F}{\partial t} + \mathbf{v} \cdot \frac{\partial F}{\partial \mathbf{r}} + \frac{\partial U}{\partial \mathbf{r}} \cdot \frac{\partial F}{\partial \mathbf{v}} = 0$$

- The force is given by

$$\frac{\partial U}{\partial \mathbf{r}} = -\frac{e}{m} \left(\mathbf{E_0} \sin\left(\omega_0 t\right) - \nabla \Phi \right) - \nu_{\mathrm{sc}} \mathbf{V}$$

- The potential is determined by

$$\nabla^2 \Phi = 4\pi \left(n_0 e \int F d^3 v - Ze \sum_i \delta(\mathbf{r} - \mathbf{r}_i) \right)$$

Kramers-Hennenberger Frame: the Rest Frame of e- Fluid

- Vlasov-Poisson equations can be linearized in a rest frame of the e- fluid
- Transformation to the e- fluid rest frame is

$$\boldsymbol{\rho} = \mathbf{r} + \frac{\boldsymbol{\epsilon}}{1 + (\nu_{sc}/\omega_0)^2}\left[\sin(\omega_0 t) + \frac{\nu_{sc}}{\omega_0}\cos(\omega_0 t)\right]$$

$$\mathbf{u} = \mathbf{v} + \frac{\omega_0 \boldsymbol{\epsilon}}{1 + (\nu_{sc}/\omega_0)^2}\left[\cos(\omega_0 t) - \frac{\nu_{sc}}{\omega_0}\sin(\omega_0 t)\right]$$

- $\boldsymbol{\epsilon} \equiv -e\mathbf{E_0}/m\omega_0^2$
- In this frame $F \approx f_0 + f$ and $\Phi = \phi_0 + \phi$

Vlasov-Poisson Equations in the e- Fluid Rest Frame

- Linearized VP equations for e- including the effect of friction with ions

$$\frac{\partial f}{\partial t} + \mathbf{u} \cdot \frac{\partial f}{\partial \boldsymbol{\rho}} + \frac{e}{m} \frac{\partial \phi}{\partial \boldsymbol{\rho}} \cdot \frac{\partial f_0}{\partial \mathbf{u}} = 0$$

$$\nabla^2_{\boldsymbol{\rho}} \phi = 4\pi e \left\{ n_0 \int \mathbf{f} d^3 u - Z \sum_i \delta \left(\boldsymbol{\rho} - \boldsymbol{\epsilon} \frac{\left[\sin(\omega_0 t) + \overbrace{(\nu_{sc}/\omega_0) \cos(\omega_0 t)}^{\text{Effect of the e-i friction}} \right]}{1 + (\nu_{sc}/\omega_0)^2} - \mathbf{r}_i \right) \right\}$$

Solution of Vlasov-Poisson Equations

- ▷ Solution is: $\phi(\mathbf{k},\omega) = S(\mathbf{k},\omega)/D(\mathbf{k},\omega)$
- ▷ Ion source term $S(\mathbf{k},\omega)$ is (D&O)

$$S(\mathbf{k},\omega) = \sum_{n,m}^{\infty} \frac{-Ze(-1)^n(-i)^m}{2\pi^2 k^2} J_n\left(\frac{\mathbf{k}\cdot\boldsymbol{\epsilon}}{1+(\nu_{sc}/\omega_0)^2}\right) \times J_m\left(\frac{\mathbf{k}\cdot\boldsymbol{\epsilon}(\nu_{sc}/\omega_0)}{1+(\nu_{sc}/\omega_0)^2}\right)\delta\left(\omega+(n+m)\omega_0\right)\sum_j e^{-i\mathbf{k}\cdot\mathbf{r}_j}$$

- ▷ $D(\mathbf{k},\omega)$ is the Lindhard dielectric function (RPA in QM treatment)

$$D(\mathbf{k},\omega) = 1 + \frac{\omega_p^2}{k^2}\lim_{\varepsilon\to 0}\int d^3\mathbf{u}\,\frac{\mathbf{k}\cdot(\partial f_0/\partial\mathbf{u})}{\omega-\mathbf{k}\cdot\mathbf{u}+i\varepsilon}$$

The Total Absorption Rate

▷ Substituting the polarization field yields the absorption rate

$$\frac{\nu_{ei}}{\omega_p} = \frac{\nu_{\rm sc}\gamma}{\omega_p} - \sum_{n,m=-\infty}^{\infty} \alpha \int d^3\mathbf{k}\, \frac{i}{k^2} \frac{J_n\left(\gamma \mathbf{k}\cdot\epsilon\right) J_m\left(\gamma \bar{\nu}_{\rm sc} \mathbf{k}\cdot\epsilon\right)}{D(\mathbf{k},(n+m)\omega_0)} \times \sum_{s=-\infty}^{\infty} (i)^s (n-s) J_{m+s}\left(\gamma \bar{\nu}_{\rm sc} \mathbf{k}\cdot\epsilon\right) J_{n-s}\left(\gamma \mathbf{k}\cdot\epsilon\right) S_{ii}(\mathbf{k})$$

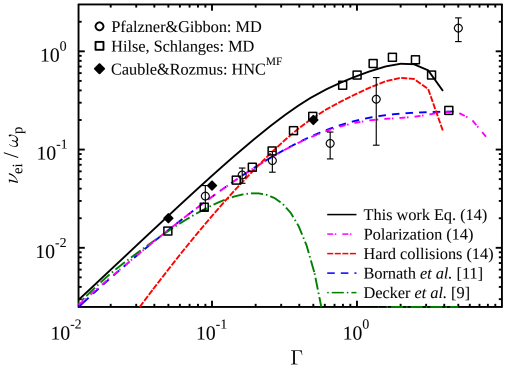
◇ $\alpha \equiv (2\pi^2)^{-1}(\omega_0/\omega_p)(Ze^2/mv_0^2)$

◇ $\gamma \equiv 1/(1+\bar{\nu}_{\rm sc}^2)$

◇ $\bar{\nu}_{\rm sc} \equiv \nu_{\rm sc}/\omega_0$

▷ Similar to Decker *et al.*, but including the e-i friction $\nu_{\rm sc} = R/m$

Results: Γ Dependence



4 S. Pfalzner, P. Gibbon, Phys. Rev. E **57**, 4698 (1998).

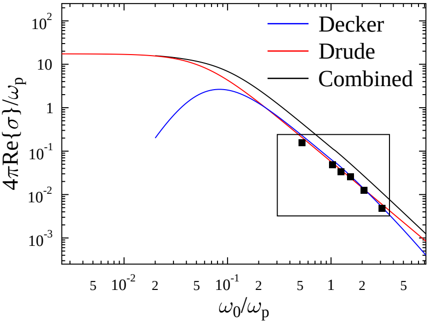
5 R. Cauble, W. Rozmus, Phys. Fluids **28**, 3387 (1985).

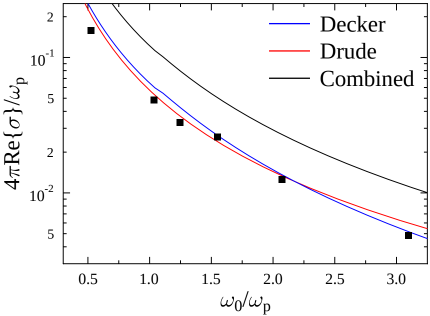
6 P. Hilse *et al.*, Phys. Rev. E **71**, 056408 (2005).

Collision frequency vs. Γ: $n_e = 10^{22}$ cm^{-3}, $Z = 1$, $v_0 = 0.2 v_{th}$, $\omega_0 = 3\omega_p$

- At $\Gamma \ll 1$ D&O/Decker limit recovered
- Using an exact form of the RPA dielectric function removes the artificial cutoff
- Good agreement with MD simulations at high Γ

Results: Frequency Dependence





Conductivity vs. ω_0: $n_e = 10^{22}$ cm^{-3}, $\Gamma = 0.15$, $v_0 = 0.2v_{th}$, $\omega_0 = 3\omega_p$

- At $\omega_0 \ll 1$ Drude limit recovered
- At $\omega_0 \gg 1$ Drude contribution vanishes slower than the Decker term !!!
- Binary collisions contribute a constant ν_{sc} to the absorption rate at $\omega_0 \gg 1$

Conclusions

- The absorption of laser energy by inverse bremsstrahlung in dense plasmas is calculated
- Effect of binary scattering is introduced using the friction force
- Mean field & binary scattering contributions treated simultaneously
- Drude & Decker results recovered in low & high frequency limits
- Results in good agreement with MD simulations obtained