Velocity distributions and kinetic equations for plasmas including Levy-type noise

- Werner Ebeling* , M.Yu. Romanovsky**, I.M. Sokolov*
- **Institute of Physics, Humboldt-University Berlin*
  ***General Physics Institute, RAS, Moscow*

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Our task

- Theoreticians like Gaussian distributions and in particular the Maxwell distribution of velocities which was found already in1866.
- In nature we find sometimes weak deviations (noneq gases and plasmas) and sometimes big deviations from Maxwell (high energy events, explosions of clusters or heavy nuclei, shocks, TOKAMAKs, laser-radiated D-clusters,.....)
- problem of hydrodynamics (E.Son)

Open questions

- Sometimes we have no good theory but assume hypothetically strong deviations as
- gas discharges (Dryuvestein distributions)
- anomal diffusion at the edge of Tokamak devices
- flow problems, ...
- The high neutron emission from irradiated clusters as due to high energy wings???

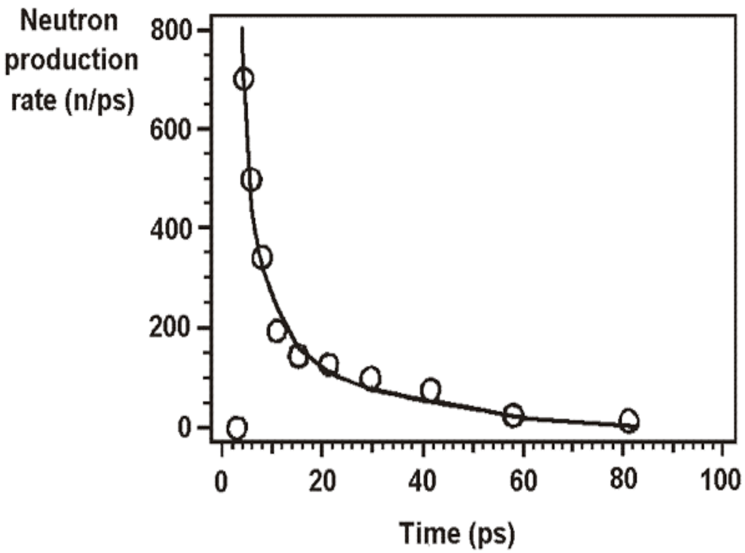
What is known about Coulomb clusters exc by fs laser beams

- See reviews Krainov/Smirnow, Phys. Reports 370, 237 (2002), + KSS Uspekhi 50, 237 (2007)

- ---10^{-14}---10^{-13} ----10^{-12} ---- 10^{-11}----
- laser pulse cluster life time formation of uniform plasma

- Coulomb energy plays a large role R(t) ~ c t (c~sound speed)
- Note: our estimate of R(t) is more complicated but leads also to a quasilinear dependence

About the LLNL EXPERIMENTS
PRL **85**, 3640 (2000)



- Laser pulse filament
- diam. 200 μm, length 1 mm
- average ion density:
  - $2 \cdot 10^{19}$ /cm³
- peak average ion energy ~
  - 12 keV
- Total neutron yield per shot ~ 5000-20000

Outline of this talk

- We explain the difference between Gauss and Levy distributions
- we study convolutions of Gauss and Levy distributions,
- we solve the Langevin equation with pure Levy and mixed noise sources,
- we discuss the high energy tails and possible applications

What is a Levy distribution?

$$W_L(\beta,\alpha) = \int_0^\infty \cos(\beta t)\exp(-t^\alpha)dt,$$

$$W_L(\beta,\alpha) \sim 1/\beta^{\alpha+1} \qquad \text{if } \alpha < 2$$

$$W_L(\beta,2) \sim \exp(-c\beta^2) \qquad \text{Gauss distr}$$

$$W_L(\beta,1) \sim \frac{a}{b+\beta^2} \qquad \text{Cauchy distr}$$

- Levy distributions are quite general distr which contain Gauss- and Cauchy- distributions as special cases. Specific property = long tails
- Note: in 3d we have a (t sin t) instead of (cos t)

Levy distr play a role in plasma physics since 1919 when Holtsmark showed that the 3d-microfield distributions are Levy-type (index 1.5)

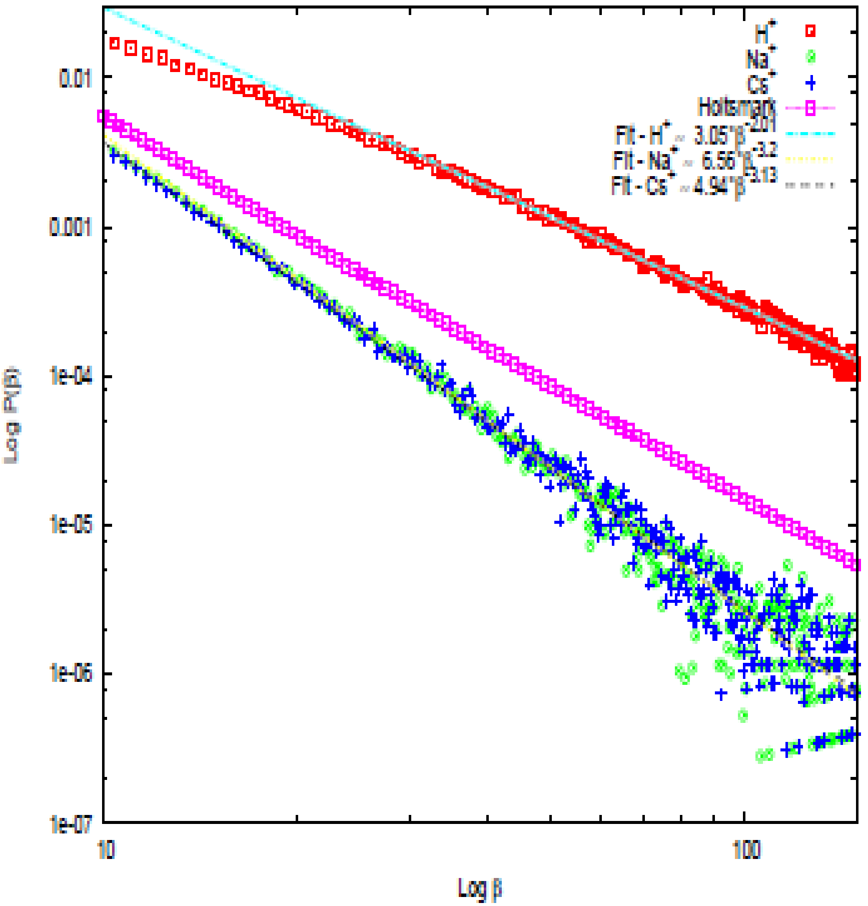
$$W_{MF}(\beta,\alpha) = C\int_0^{\infty} t\sin(\beta t)\exp(-t^{\alpha})dt,$$

$$W_{MF}(\beta,\alpha) \sim 1/\beta^{\alpha+1}, \ldots. E_H = 1.2en^{2/3}$$

- Here beta = E / E_n
- E_n charactristic field (E_H - Holtsmark field)
- alpha = 1.5 for Holtsmark distributions
- Mayer/Broyles fould alpha =2 for dense plasmas
- MRom found alpha = 1 for Kepler scattering
- Our simul: index changes in range 0.5<alpha<1.5,

Levy exponents of the tails from simulations (Sadykova/Valuev workshop 2009)

- In the present example Gamma=2 we find
- alpha < 1 for H
- alpha ~1.5 for neutrals
- alpha ~ 2 for alkali
- Microfield distributions are appr of Levy-type but 1<alpha<2 and E_n are free parameters, depending on species, n, T, the main body is Gaussian



Here we are interested not in Microfield distributions but in Velocity Distributions which are generated by microscopic stochastic forces: Gausian collisions + Holtsm-electr microfields.

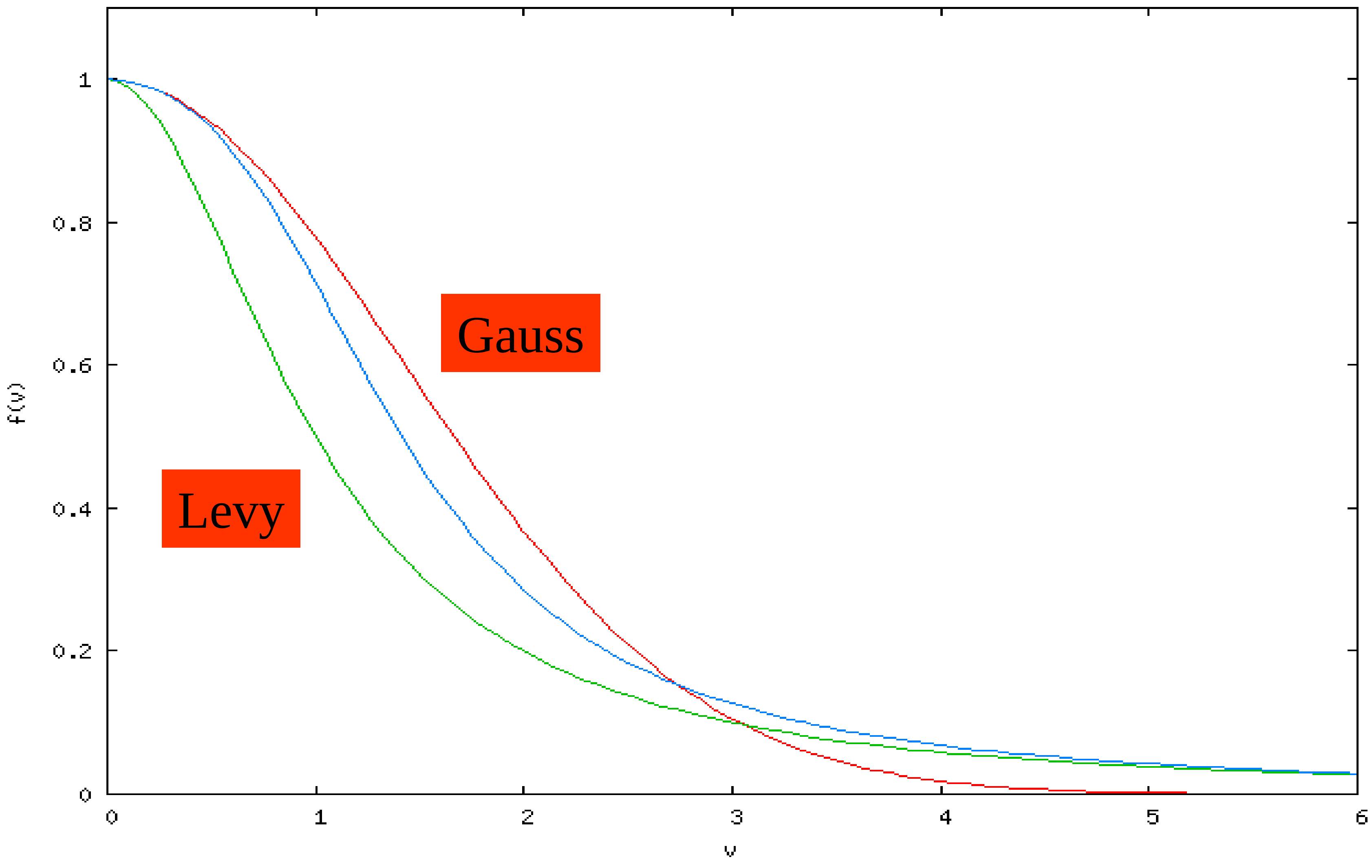
- Collisions corr to Gauss-distributed forces, microfields to Levy-distributed forces. In a first appr we obtain the integral distribution by a convolution of Gaussian with Levy-distributions

$$W(y) = \int dz W_G(y-z) W_L(z)$$

$$W(y) = \int_{-\infty}^{\infty} dk \cos(ky) \exp(-at^{\alpha} - bt^2)$$

$$W(y) \sim 1/y^{\alpha+1}$$

Comparison of Gauss- with Cauchy distr and convoluted Gauss-Cauchy distr



For Gaussian stoch forces follows the standard Fokker-Planck eq.

(a special case is the Landau eq.),

Maxwell solutions

$$\frac{\partial}{\partial t} f(v,t) = \nabla[\gamma\, v f(v,t)] + D\nabla^2 f(v,t)$$

$$f_0(v) = C\exp\left[-\frac{\gamma v^2}{2D}\right] = C\exp\left[-\frac{mv^2}{2k_B T}\right]$$

For Levy forces follows a kinetic equation with fractal derivatives

$$\frac{\partial}{\partial t} f(v,t) = \nabla[\gamma\, v f(v,t)] + D\nabla^{\alpha} f(v,t)$$

$$\frac{\partial}{\partial t} f(k,t) = -\gamma\, k \frac{\partial}{\partial k} f(k,t) + D k^{\alpha} f(k,t)$$

The velocity distribution function for Levy noise

$$\frac{\partial}{\partial t} f(k,t) = -\gamma_0 k \frac{\partial}{\partial k} f(k,t) + Dk^{\alpha} f(k,t)$$

$$f_0(k) = \exp\left[- D(t)\,|\,k\,|^{\alpha}\right]; D(t) = \frac{qE_n}{m\alpha\gamma_0}[1 - \exp(-\alpha\gamma_0 t)]$$

$$W(|\,v\,|) = \frac{2\,|\,v\,|}{\pi}\int_0^{\infty} t\sin(|\,v\,|\,t)\exp(-t^{\alpha})dt \sim \frac{d(t)}{\alpha\,|\,v\,|^{\alpha+1}}$$

$$\alpha = 1 \longrightarrow W\,(|\,v\,|) = \frac{4d(t)\,|\,v\,|^2}{\pi[d(t) + |\,v\,|^2\,]^2}$$

The velocity distribution for Gauss+Levy forces

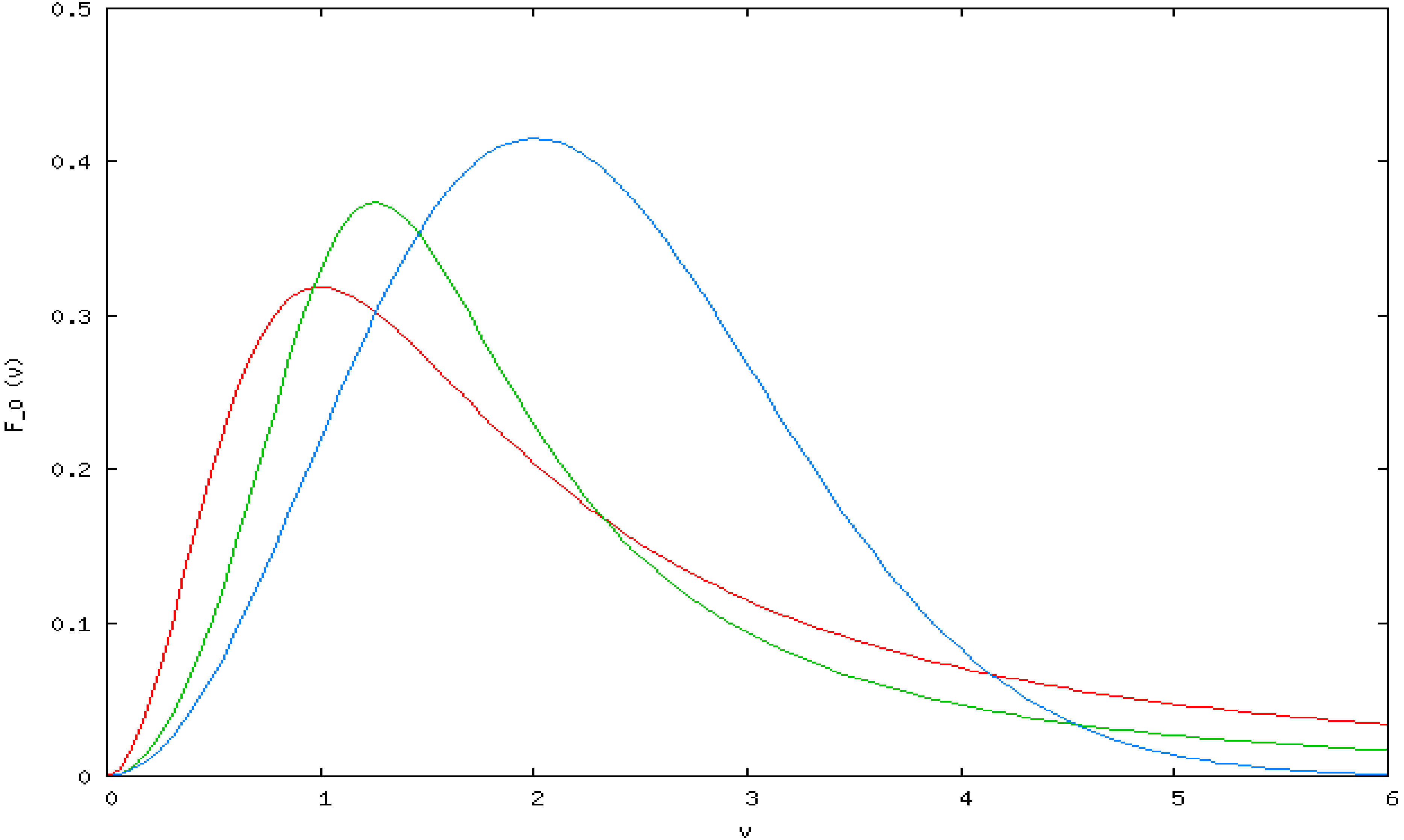
$$\frac{\partial}{\partial t} f(k,t) = -\gamma_0 k \frac{\partial}{\partial k} f(k,t) + (D_\alpha k^\alpha + D_2 k^2) f(k,t)$$

$$f_0(k) = \exp\left[-d_\alpha(t) D_\alpha |k|^\alpha - d_2(t) D_2 k^2\right];$$

$$d_\alpha(t) = \frac{1}{\alpha\gamma}[1 - \exp(-\alpha\gamma\, t)]$$

$$W(|v|) = \frac{2|v|}{\pi} \int_0^\infty kv \sin(k|v|) \exp\left(-\frac{D_\alpha |k|^\alpha}{\alpha\gamma} - \frac{D_2 k^2}{2\gamma}\right) dt \sim \frac{d(t)}{\alpha |v|^{\alpha+1}}$$

Distr of the modulus of the velocity (root of kinetic energy)



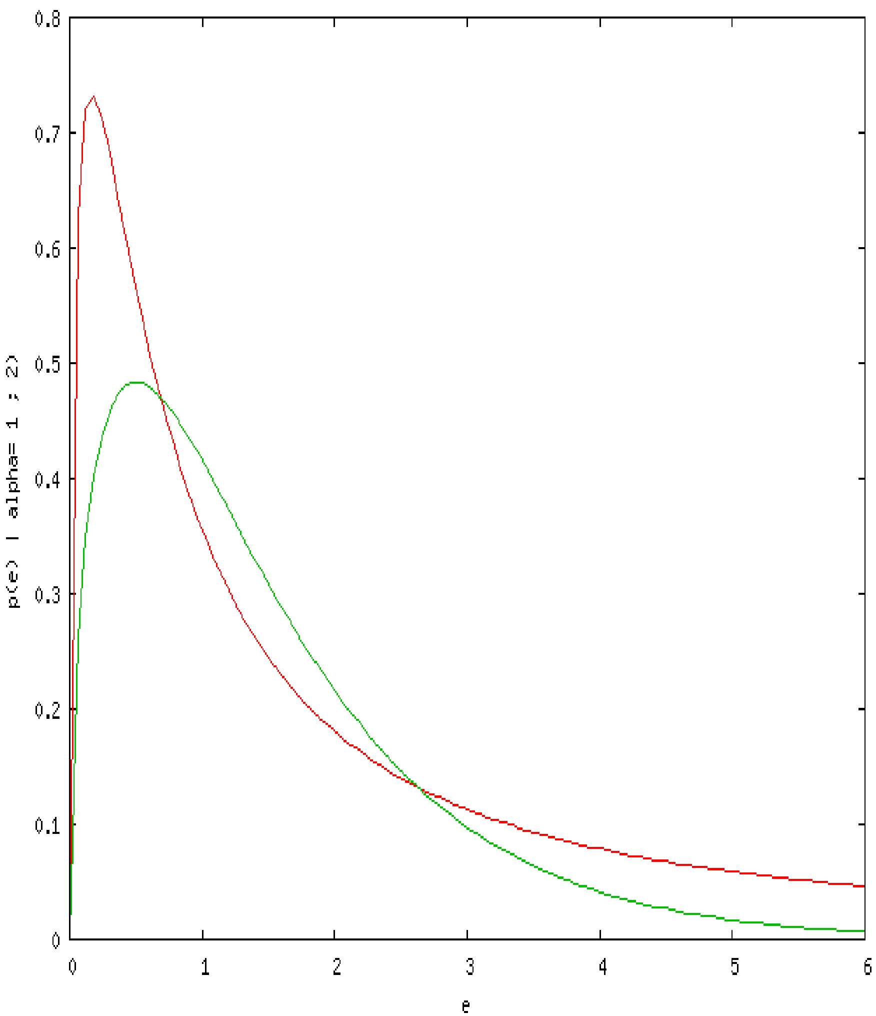
The energy distribution for Gauss+Levy forces

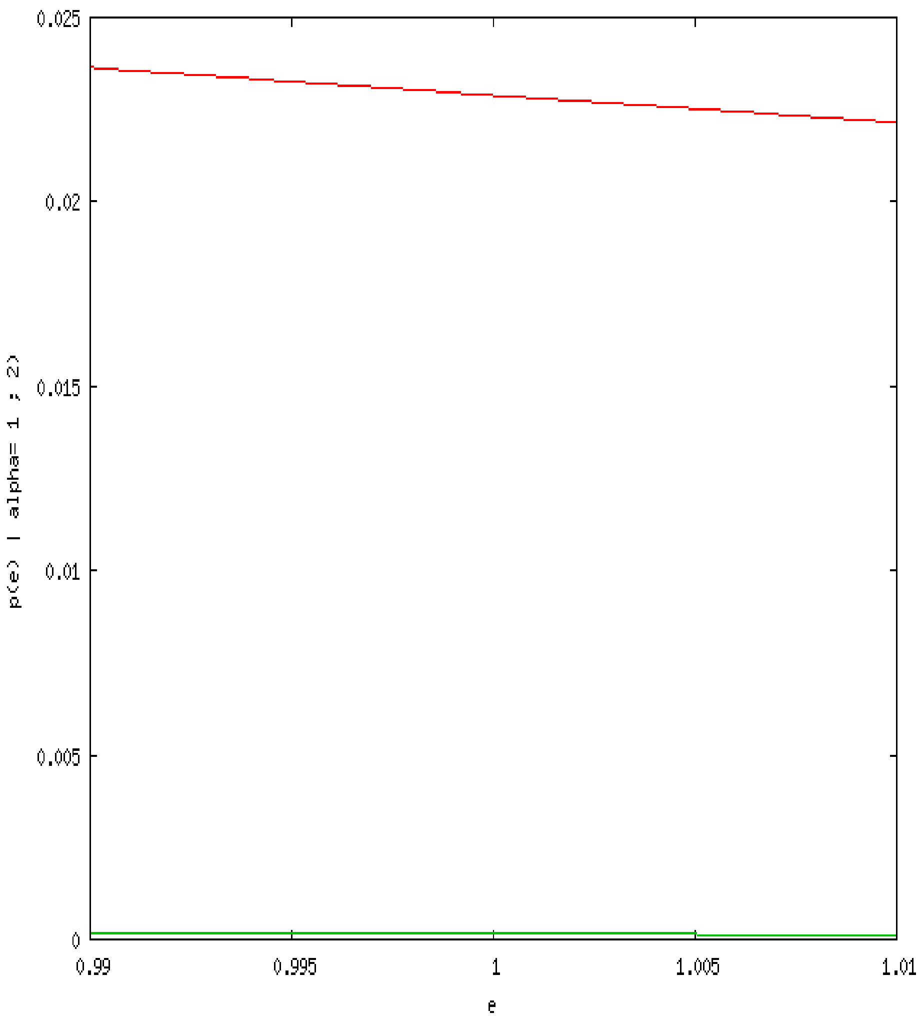
$$W(\varepsilon) = C\int_0^\infty k\sqrt{2m\varepsilon}\,\sin(k\sqrt{2m\varepsilon})\exp\left(-\frac{D_\alpha k^\alpha}{\alpha\gamma} - \frac{D_2 k^2}{2\gamma}\right)dk$$

$$W(\varepsilon) \sim \frac{const}{\alpha\,\varepsilon^{\alpha+1/2}}, \quad \mathrm{W}(\varepsilon > \varepsilon_0) \sim \frac{const}{\alpha\,(\alpha+1)\varepsilon^{\alpha-1/2}}$$

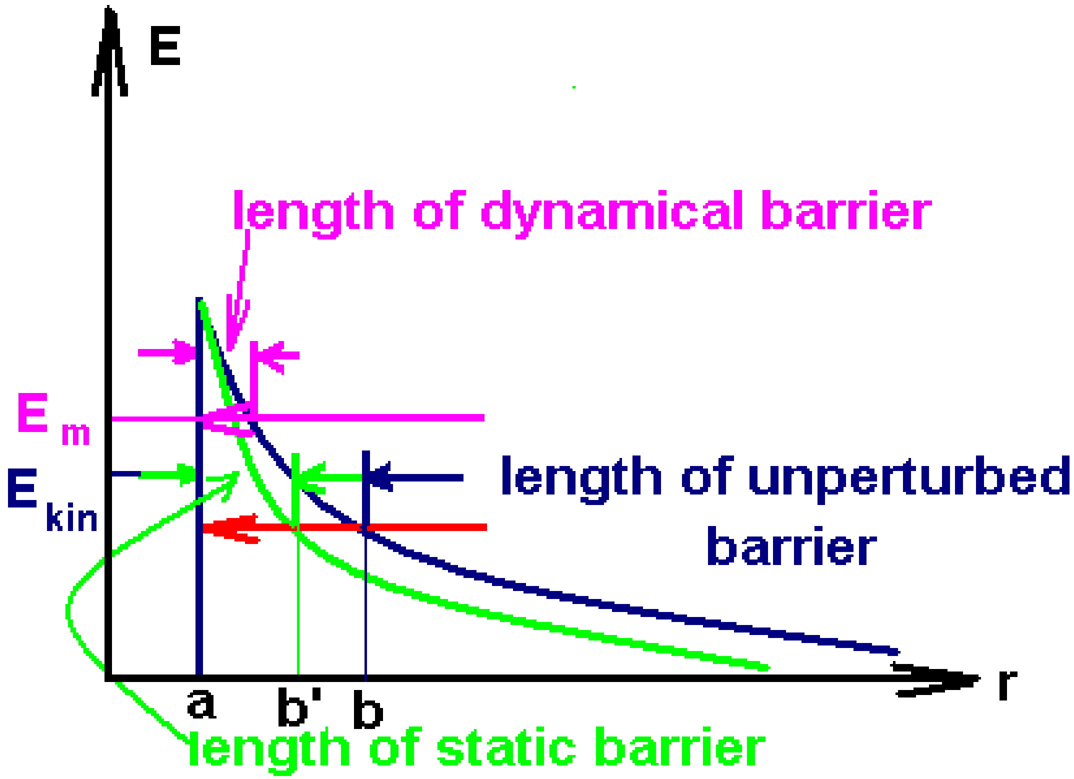
Difference between Gauss and Cauchy distr

is for (E/kBT ~ 10) already about 140, increases quickly





The problem of nuclear fusion (Gamov model)



- Fusion occurs as the result of tunneling of protons/deuterons through the Coulombic barrier: Enhancement requires influencing the barrier during the reaction time
- **or changing velo-city distribution**

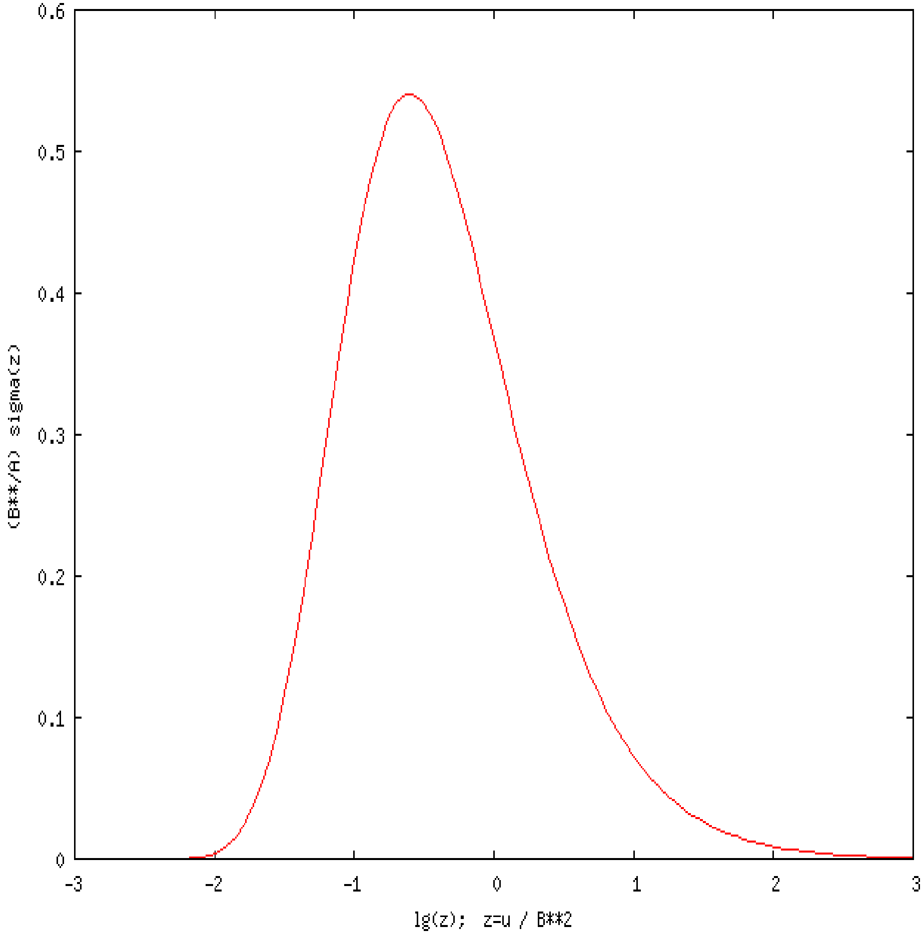
Take Thompson cross section of fusion average over energy distr (Cauchy)

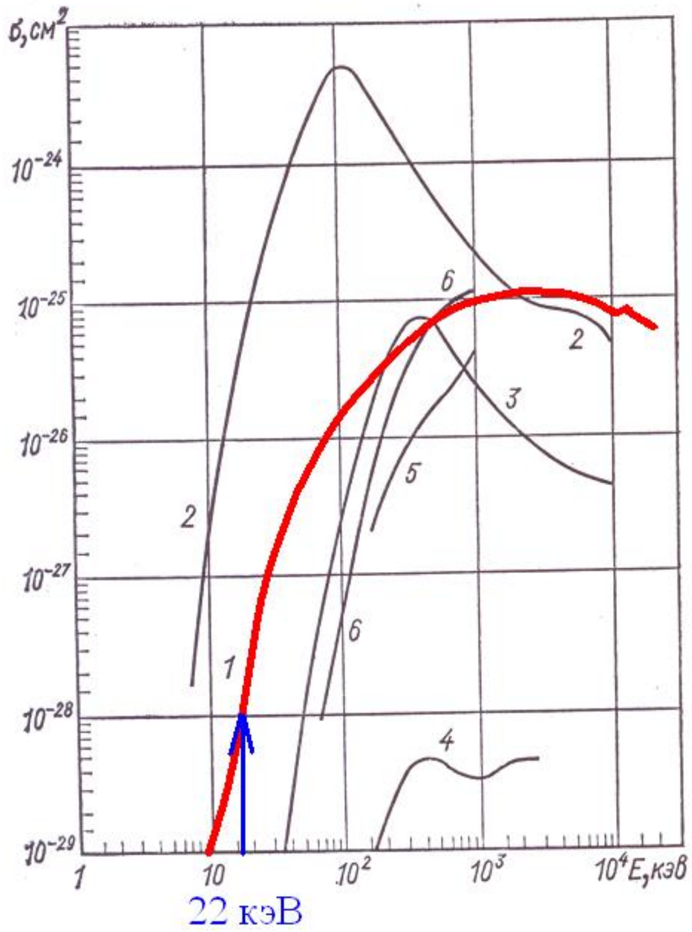
$$\sigma(U) = \frac{A}{U}\exp(-\frac{B}{\sqrt{U}})$$

$$\frac{dN_n}{dt} = Nn(t)v\sigma(u)$$

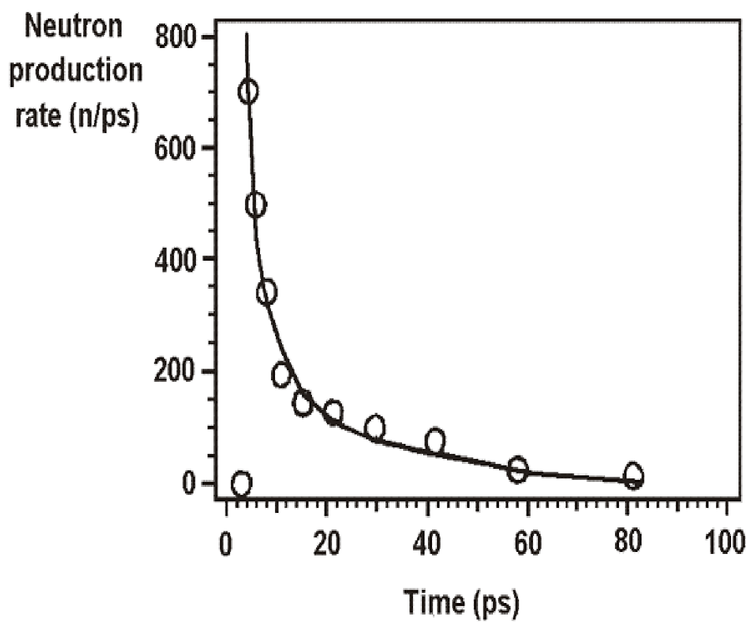
$$\frac{dN_n}{dt} = Nn(t)\frac{2Ad(t)}{\pi B^2}\int_0^\infty du\exp[-\frac{B}{\sqrt{u}}]\frac{B^2}{[u+d(t)(m\ /2)]^2}$$

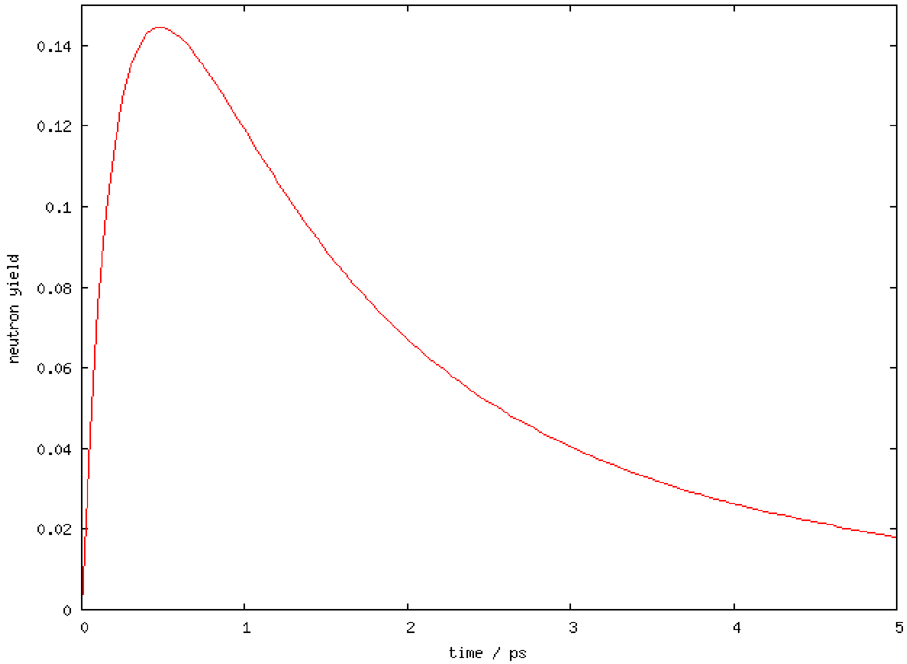
The Thompson semi-empirical
REACTION CROSS-SECTION
left: Thompson formula;   right:data for several reactions DD=red





LLNL EXPERIMENTS (PRL **85**, 3640 (2000)) vs. our estimate





Exp: Shot about 50 ps

average ion density:

- 2•10 ¹9⁹ cm⁻³

peak average ion energy

- ~ 12 keV

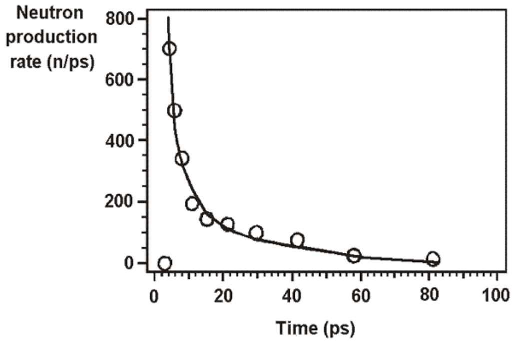
Total neutron yield

100-1000 neutrons/shot

- estimate ~ 30-100

Interpretation: Fast ionization of clusters by strong laser pulses, explosion due to Coulomb repulsion

- in subsequent collisions sometimes very high fields are created which accelerate D-D fusion
- LLNL-group: main role play "collisions of clusters" PRL **85**, 3640 (20
- Our estimate: coll times too short cross sections too small. Th the observation of neutrons 100 -1000 neutrons / ps remains unexplained



Our interpretation

- The strong repulsive forces in Coulomb clusters lead to deviations from Maxwell distributions with power law decay.
- Short time high energy wings appear in the range 10 fs - 100 fs decay within 1 ps.
- High energetic protons/deuterons may penetrate the fusion barrier
- Theoretical approach: study the velocity distributions = Levy distributions with long tails.
- enhancement of fusion rates due to the long tails in the velocity distribution

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