Low Velocity Ion Slowing Down in a Strongly Magnetized Plasma Target

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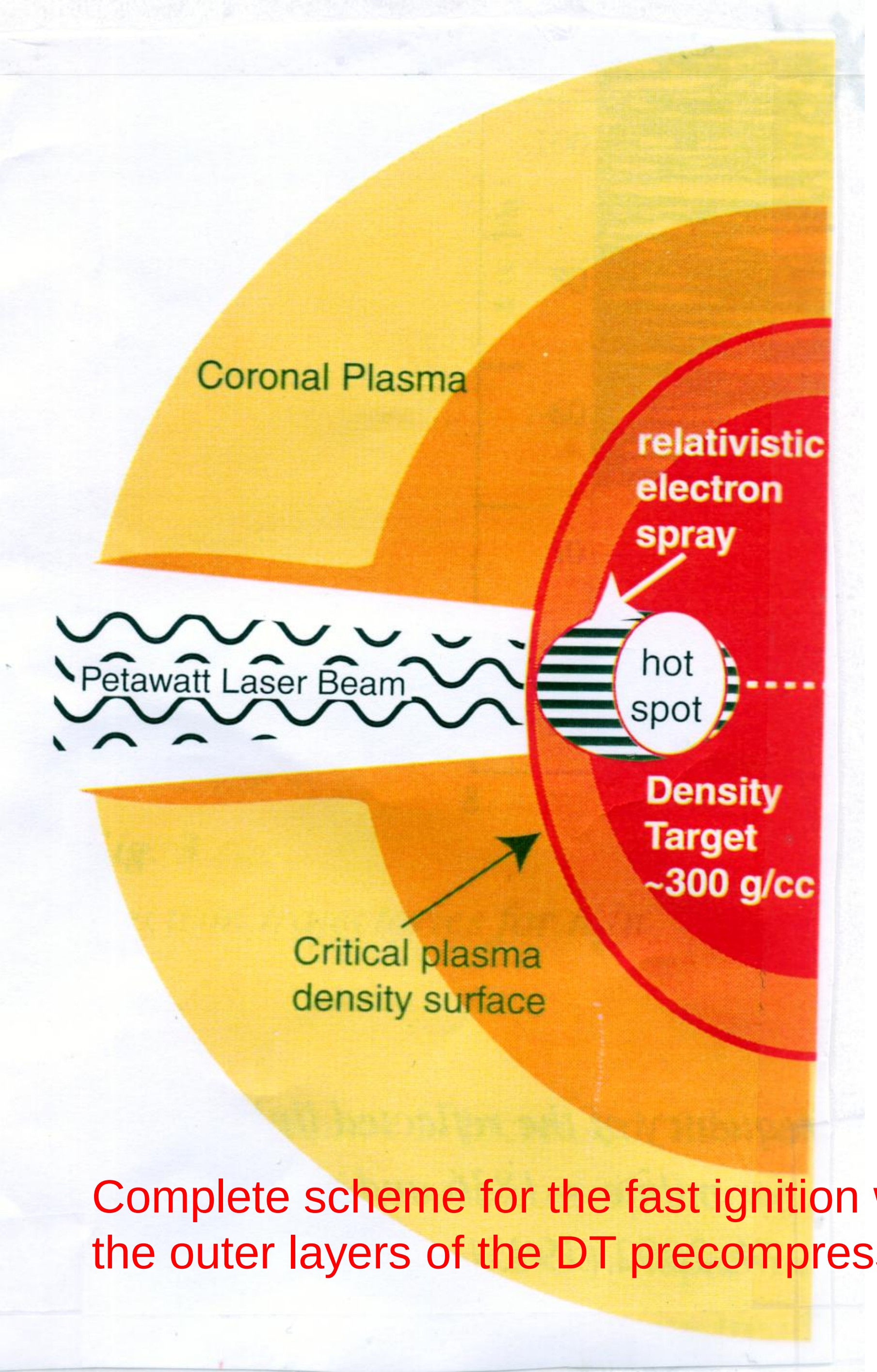
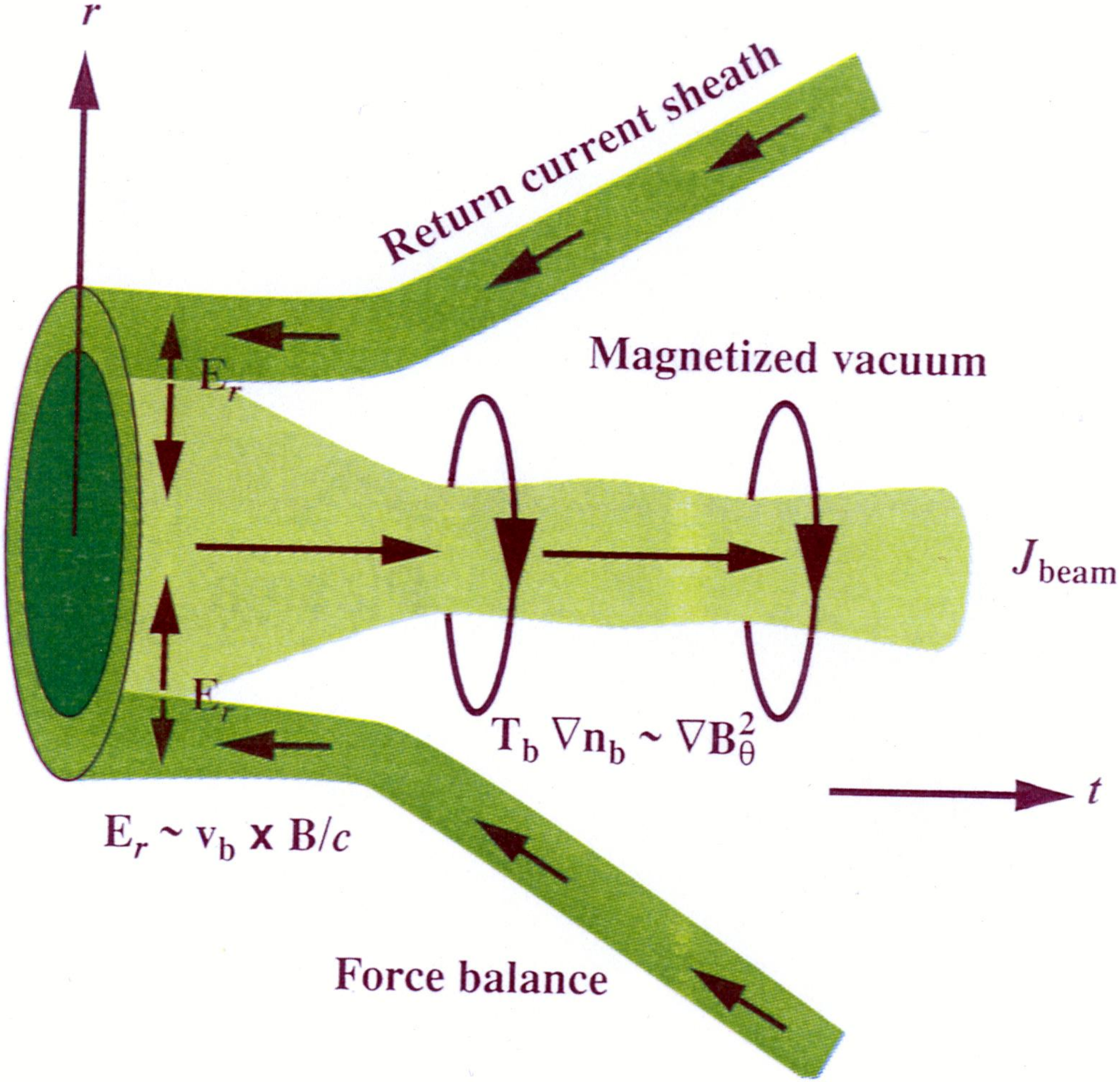


Complete scheme for the fast ignition with hole boring in the outer layers of the DT precompressed core


r
Return current sheath
Magnetized vacuum
E_r
E_r
J_{beam}
$T_b \nabla n_b \sim \nabla B_\theta^2$
t
$E_r \sim v_b \times B/c$
Force balance

The Coulomb coupling parameter for a plasma obeying the classical statistics is

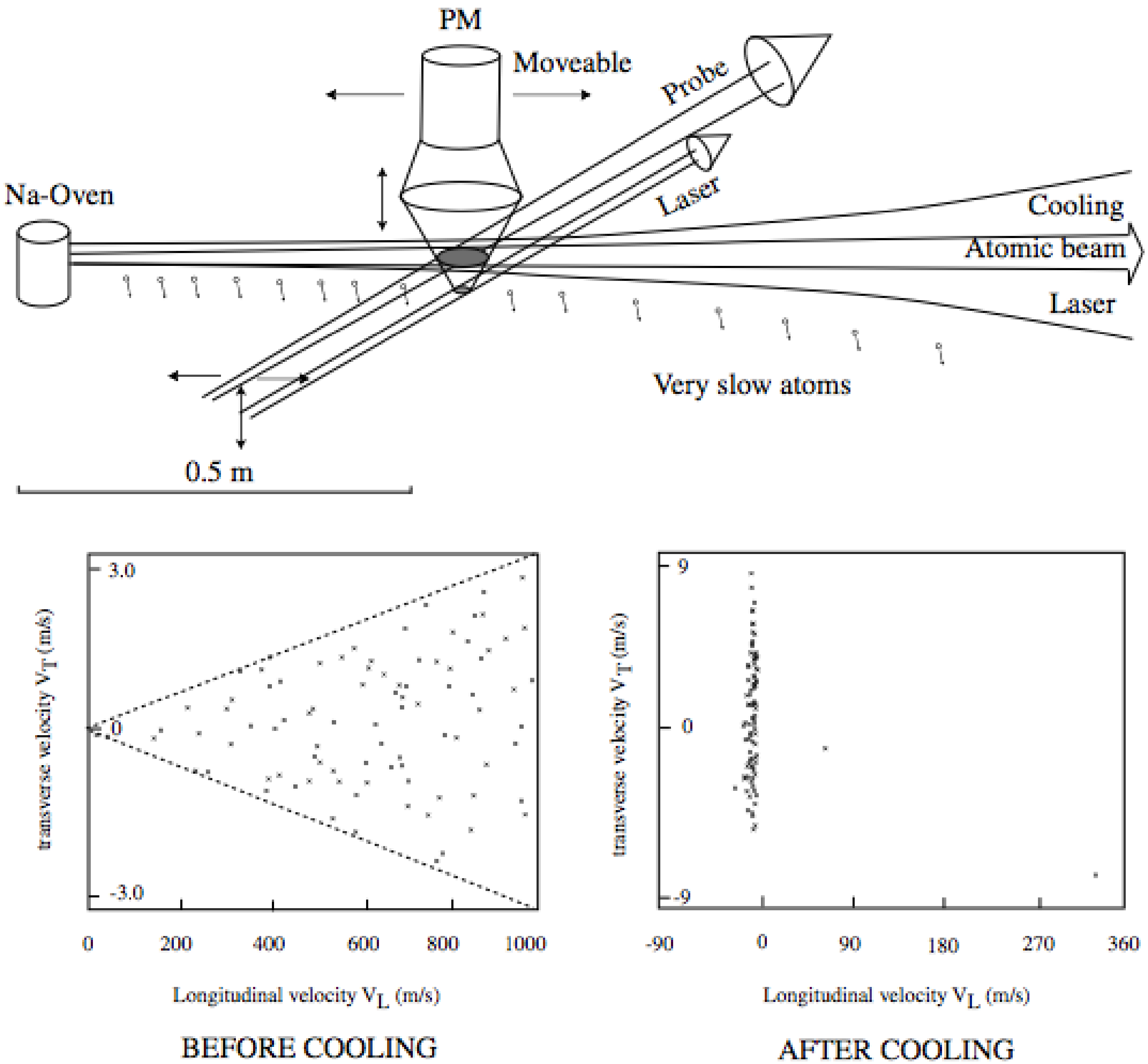
$$\Gamma_e = \frac{(Ze)^2}{aT} = 2.69 \times 10^{-5}\, Z^2 \left(\frac{n_e}{10^{12}\mathrm{cm}^{-3}}\right)^{1/3} \left(\frac{T_e}{10^6\mathrm{K}}\right)^{-1}. \quad \gtrsim 0.2$$

$$r_s = \frac{a_e}{r_B} \approx \left[\frac{n_e}{1.6 \cdot 10^{24}\mathrm{cm}^{-3}}\right]^{-1/3} \ggg 1$$

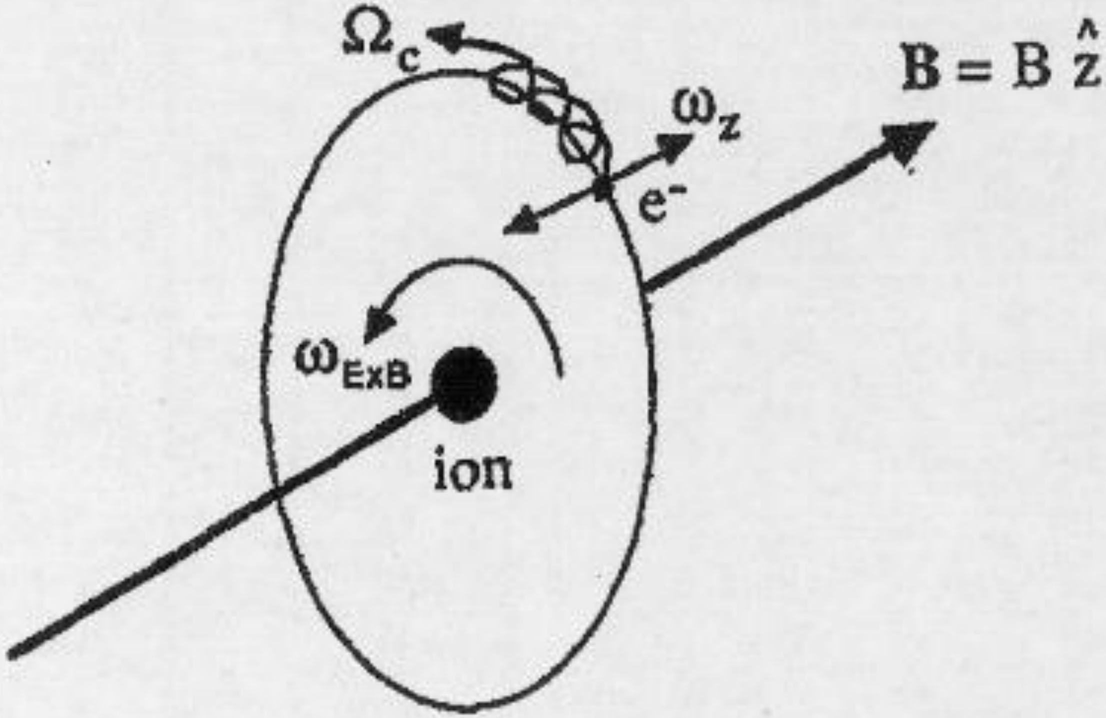
$$\Theta = \frac{k_B T_e}{E_F} = \frac{2m_e k_B T_e}{\hbar^2 k_F^2} = \frac{2m_e k_B T_e}{\hbar^2 (3\pi^2 n_e)^{2/3}}$$

$$= 4\pi \left(\frac{4}{9\pi}\right)^{2/3} \left(\frac{a_e}{\lambda_{th,e}}\right)^2 = 2\left(\frac{4}{9\pi}\right)^{2/3} \frac{r_s}{\Gamma_e} \gg 1$$

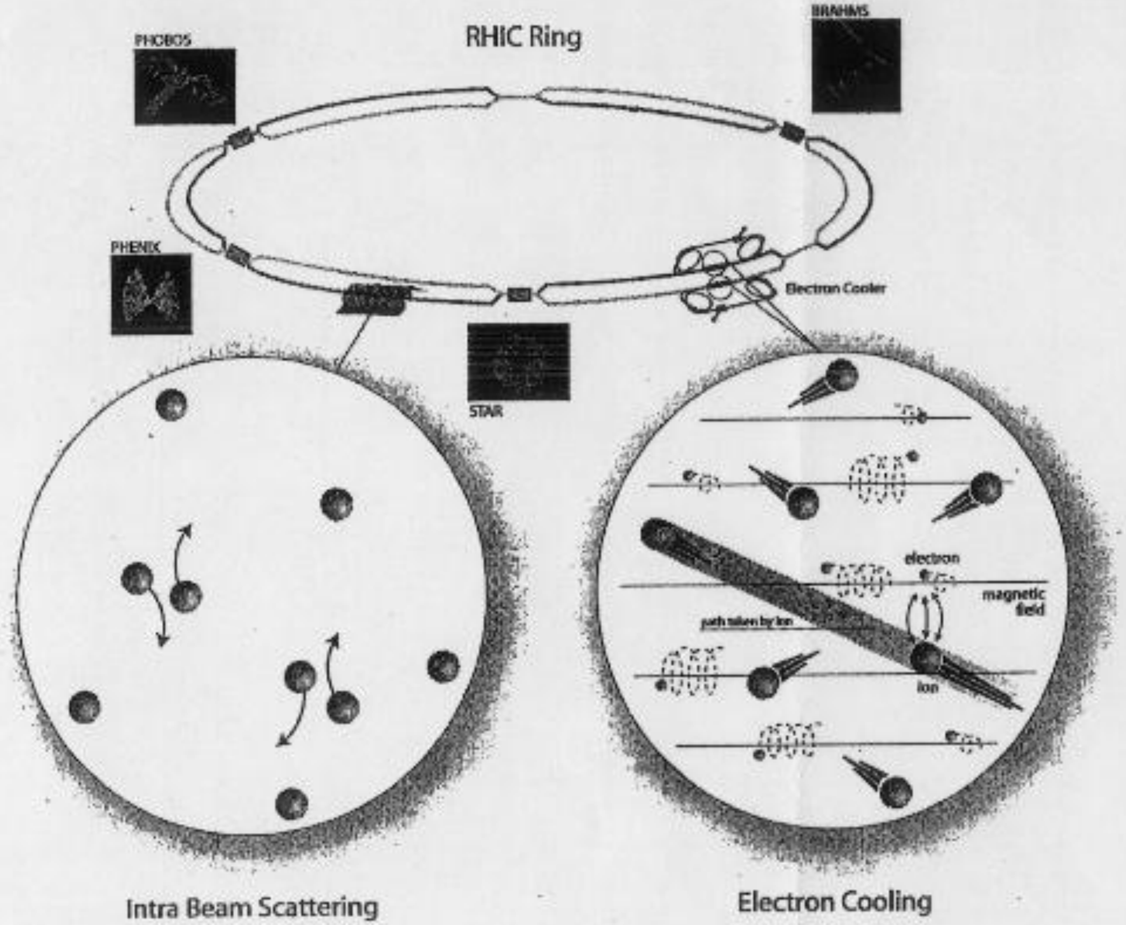
CLASSICAL ELECTRON FLUID



Particle slowing using chirped laser frequency

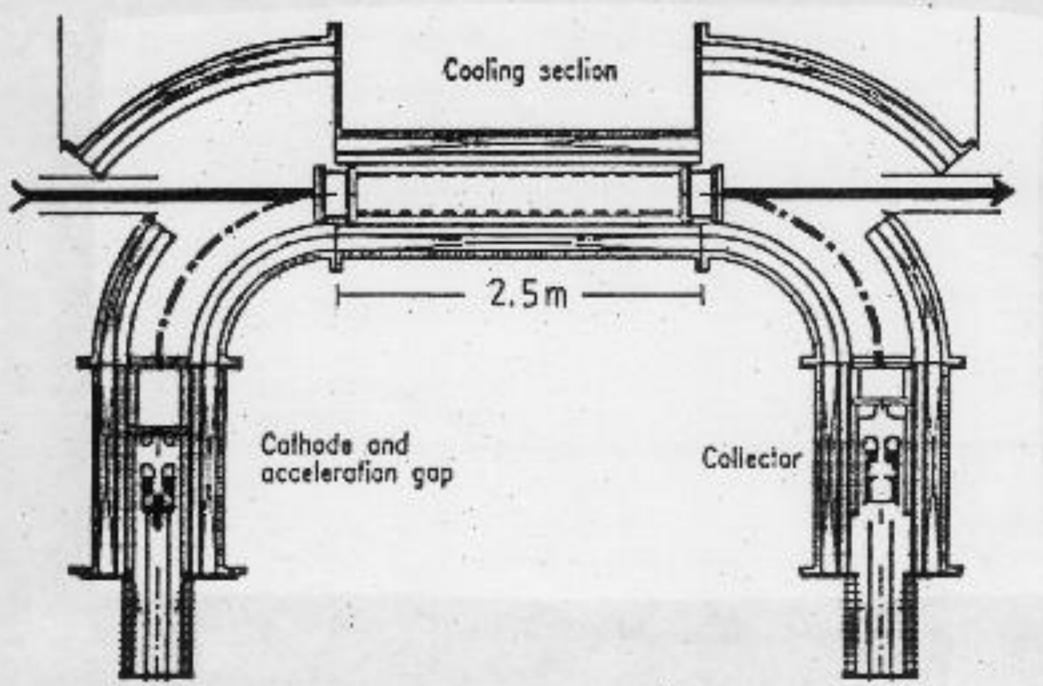


Drawing of guiding center atom. In order of descending frequency, the electron executes cyclotron motion, oscillates back and forth along a field line in the Coulomb well of the ion, and $\mathbf{E}\times\mathbf{B}$ drifts around the ion.



Electron cooling is a method to reduce the beam size in ion storage rings "Cold" electrons are used to cool the "hot" ion beam. The result of cooling is a smaller beam size, a higher particle density and therefore higher luminosity.

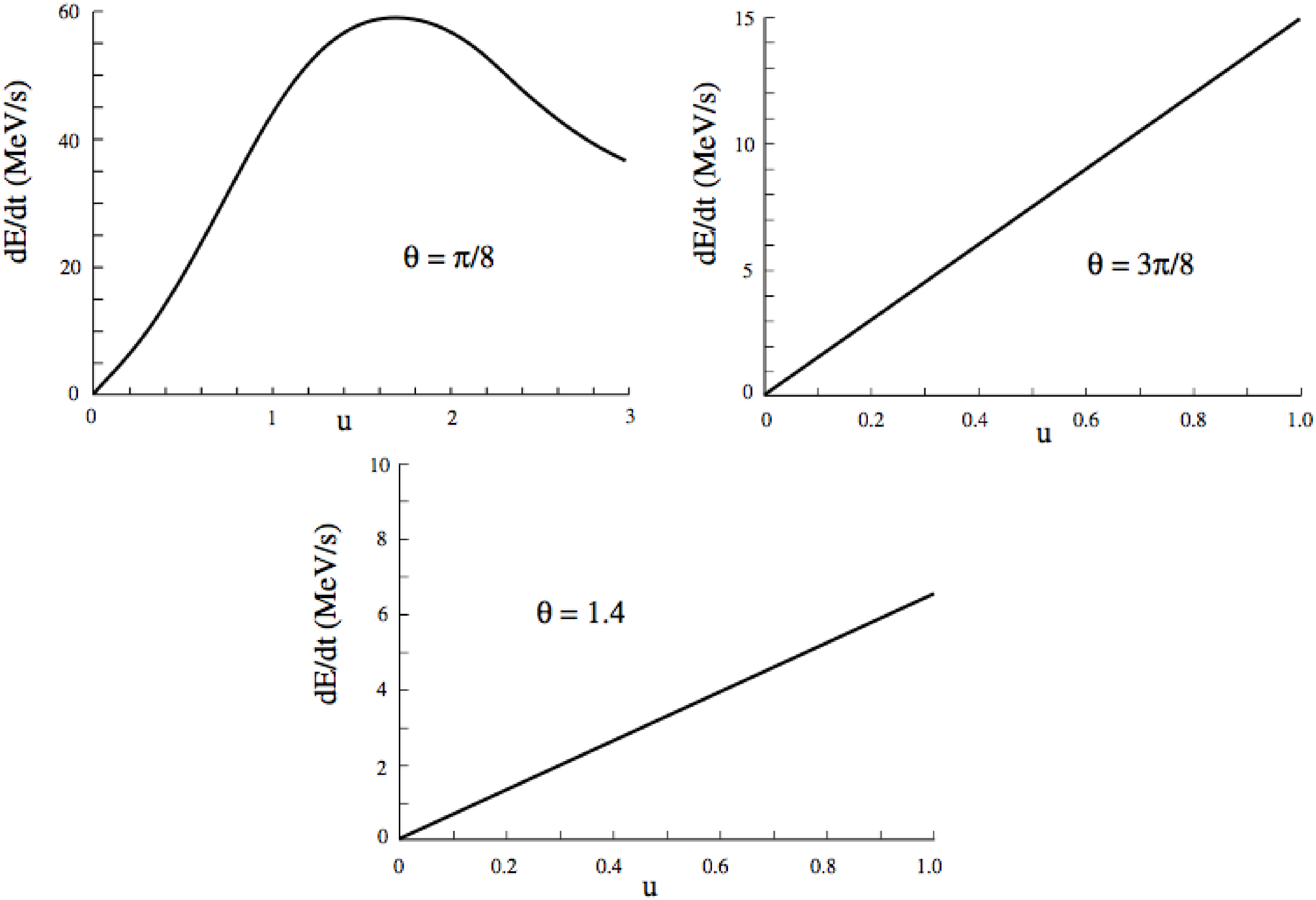
ies can be varied between 2 and 320 keV with a maximum design cur-
ent of 10 A. The e-beam diameter is 5 cm and the length of the
lectron-ion interaction region 250 cm.



Electron cooling device

Ion	C^{6+}
B (T)	0.005–0.05
Ω (s^{-1})	0.88×10^{9}–0.88×10^{10}
$\frac{(k_B T_\perp/m)^{1/2}}{\Omega}$ (m)	5.1×10^{-6}–5.1×10^{-5}
n_e (cm^{-3})	8×10^{6}
$k_B T_\perp$ (meV)	11.5
$k_B T_\parallel$ (meV)	0.1
λ_D (m)	0.73×10^{-4}

Heidelberg Test Storage Ring TSR


dE/dt (MeV/s)
u
θ = π/8
θ = 3π/8
θ = 1.4

We thus consider ion stopping

$$S(V_b) \equiv \frac{dE_b}{dx}(V_b) \qquad (1)$$

near $V_b = 0$. The ratio $S(V_b)/V_b$ usually monitors a linear stopping profile, up to 100 keV/a.m.u in cold matter. Similar trends are also reported in highly ionized plasma with $B = 0$ or $B \neq 0$.

From now on, we intend to make use of a very powerful connection between very low velocity ion stopping and particle diffusion through Einstein characterization of ion mobility associated to thermal electron fluctuations in target, around the slow ion projectile visualized as an impurity immersed in a dense and homogeneous electron fluid.

$$\lim_{V_b \to 0} \frac{S(V_b)}{V_b} = k_B T_e D^{-1} \quad , \qquad (2)$$

$D_{\perp}$ and $D_{//}$ expressions introduced in Eq. (2) are expected to document a strong anisotropy between transverse and parallel slowing down. However, in both cases, B-dependence is obviously increasing with B^2 (classical) or B (Bohmlike).

We thus consider respectively a dense and strongly magnetized target plasma envisioned for fast ignition in ICF with $n = 10^{21}$ e-cm^{-3}, T = 1 keV and $10^{10} \leq B(G) < 10^{11}$ and also a highly dilute ($n \sim 10^7$ e-cm^{-3}) one at very low temperature (T (° K) = 100), of current use for ion beam cooling, on the LEAR accelerating line, at CERN for instance. In both cases, one witnesses a steady LVISD increase with B, for both $D_{\perp}$ and $D_{//}$, when B-dependent D expressions are introduced in Eq. (2).

In a magnetized plasma the particle self-diffusion coefficient D can be readily expressed in terms of Green-Kubo integrands (GKI) involving field fluctuations in the target electron fluid, under the form

$$D = \frac{c^2}{B^2} \int_0^\infty d\tau \left\langle E(\tau) \cdot E(0) \right\rangle \qquad (3)$$

References

- C.Deutsch and R.Popoff Phys.Lett.A372,5804(2008)
- Phys.Rev.E78,056405(2008)

This procedure implies that the slowly incoming ions are evolving against a background of faster fluctuating target electrons ($V_b < V_{the}$) providing the OCP rigid neutralizing background thus validating the OCP assumption.

Moreover, restricting to proton projectiles impacting an electron-proton plasma, we immediately perceive the pertinence of the diffusion-based LVISD.

First, the proton beam can easily self-diffuse amongst its target homologues, while the same mechanism experienced by target electrons allow them to drag ambipolarly the incoming proton projectiles.

So, the transverse electron LVISD can be either monitored by the well-know classical diffusion $D_\perp \sim B^{-2}$, or by the Bohmlike hydrodynamic one with $D_\perp \sim B^{-1}$.

In the first case, momentum conservation at the level of the electron-ion pair implies that the ions will diffuse with the same coefficient as the electrons. On the other hand, the hydro Bohm diffusion across **B** is operated through clumps with a large number of particles involved in this collective process.

MHD-OCP modes

Transverse D⊥ and parallel D// diffusion coefficient have already been discussed at length by Marchetti et al. and Cohen-Suttorp. Their derivation is based on the specific features of four finite frequency and propagating hydromodes in a strongly magnetized OCP with the ratio of plasma-to cyclotron-frequencies, ⌉p/⌉b<1.

According to the Marchetti-Kikpatrick-Dorfman (MKD) analysis there are five modes: four propagating finite-frequency modes and one purely diffusive mode

Here $\hat{k}_{\perp} = k_{\perp}/k$ and $\hat{k}_{Z} = k_{Z}/k$

Strongly Magnetized Modes for Transverse Propagation

- 1-Two High Frequency Modes→Bernstein (upper Hybrid) for B→0
- 2-Two Finite Frequency Modes→Propagating (shear) for B→0
- 3-One Diffusive Heat Mode.Irrelevant for Particle transport

Self-Diffusion Coefficients

So, exploring first the $\omega_b \geq \omega_p$ domain, one can explicit the parallel and B-independent diffusion

$$D_{//}^{(0)} = \frac{3\sqrt{\pi}}{M_p V_{thi}^2 \nu_c} \sim 0(\omega_b^0) \qquad (9a)$$

where $V_{thi}^2 = k_B T/M_p$, and $\nu_c = \omega_p \varepsilon_p \ell n (1/\varepsilon_p)$ in terms of the plasma parameter $\varepsilon_p = 1/n\lambda_D^3$, where n denotes charge particle density, and λ_D, the Debye length, in a beam-plasma system taken as globally neutral with $\nu_c/\omega_b << 1$.

At the same level of approximation transverse diffusion reads as

$$D_{\perp}^{(0)} = \frac{r_L^2 \nu_c}{3\sqrt{\pi}} \sim 0(\omega_b^{-2}) \ , \qquad (9b)$$

in terms of Larmor radius $r_L = V_{thi}/\omega_b$.

with higher B values ($\omega_b >> \omega_p$) one reaches the transverse hydro Bohm regime featuring

$$D_\perp = D_\perp^{(0)} + \frac{0.5 V_{thi}^2}{\omega_b} \varepsilon_p^2 \left(\ln(1/\varepsilon_p)\right)^{3/2} , \qquad (10)$$

while parallel diffusion retains a ω_b-dependence through

$$D_{//}^{-1} = \frac{\Gamma^{5/2}}{\omega_p a^2} \cdot \left(\frac{3}{\pi}\right)^{1/2} \cdot \left(0.5\, \mathrm{Log}(1+X^2) - 0.3 + \frac{0.0235}{r^2}\right), \qquad (11)$$

where $\Gamma = \frac{a^2}{3\lambda_D^2}$ with $a = \left(\frac{3}{4\pi n}\right)^{1/3}$, $r = \frac{\omega_p}{\omega_b}$ and

$$X = \frac{1}{\sqrt{3}} \cdot \frac{1}{\Gamma^{3/2}}$$

$$-\frac{dE}{dx} = \mathcal{R}_1 \upsilon_p + \mathcal{R}_3 \upsilon_p^3 + 0(\upsilon_p^5) \qquad (12)$$

with the 'friction coefficient'

$$\mathcal{R}_1 = \frac{Z^2 N_D}{12\pi\sqrt{2\pi}}\left(\ln(K^2+1) - \frac{K^2}{K^2+1}\right), \qquad (12a)$$

and the υ_p^3 coefficient

$$\mathcal{R}_3 = \frac{Z^2 N_D}{12\pi\sqrt{2\pi}}\left(-\frac{3}{10}\ln(K^2+1) + \left(\frac{8}{5} - \frac{\pi}{20}\right)\right.$$

$$-\frac{29}{10}\frac{1}{K^2+1} + \left(\frac{13}{10} + \frac{3\pi}{20}\right) \qquad (12b)$$

$$\left.\times \frac{1}{(K^2+1)^2} + \frac{\pi}{10}\frac{1}{(K^2+1)^3}\right)$$

where $Z = Z/N_D$ and $K = 8\pi n \lambda^3{}_D/Z$.

In the present $\varepsilon_p < 1$ approximation, we always witness $\mathcal{R}_3 \lll \mathcal{R}_1$, which validates quantitatively the exact DB result (2).

An obvious alternative to the series (12) is afforded through the B = 0 limit of the B-perturbative analysis worked out by Steinberg and Ortner, with

$$-\frac{dE}{dx} = \frac{2}{3}\frac{(2\pi m)^{1/2}}{(kT)^{3/2}} Z^2 e^4 n\, DV_p \qquad (13)$$

m = electron mass, and $D = -\ln(y^2/8) - C - 1$,

where $y = \hbar\omega_p/kT$

and C = 0.5722, denotes the Euler constant.

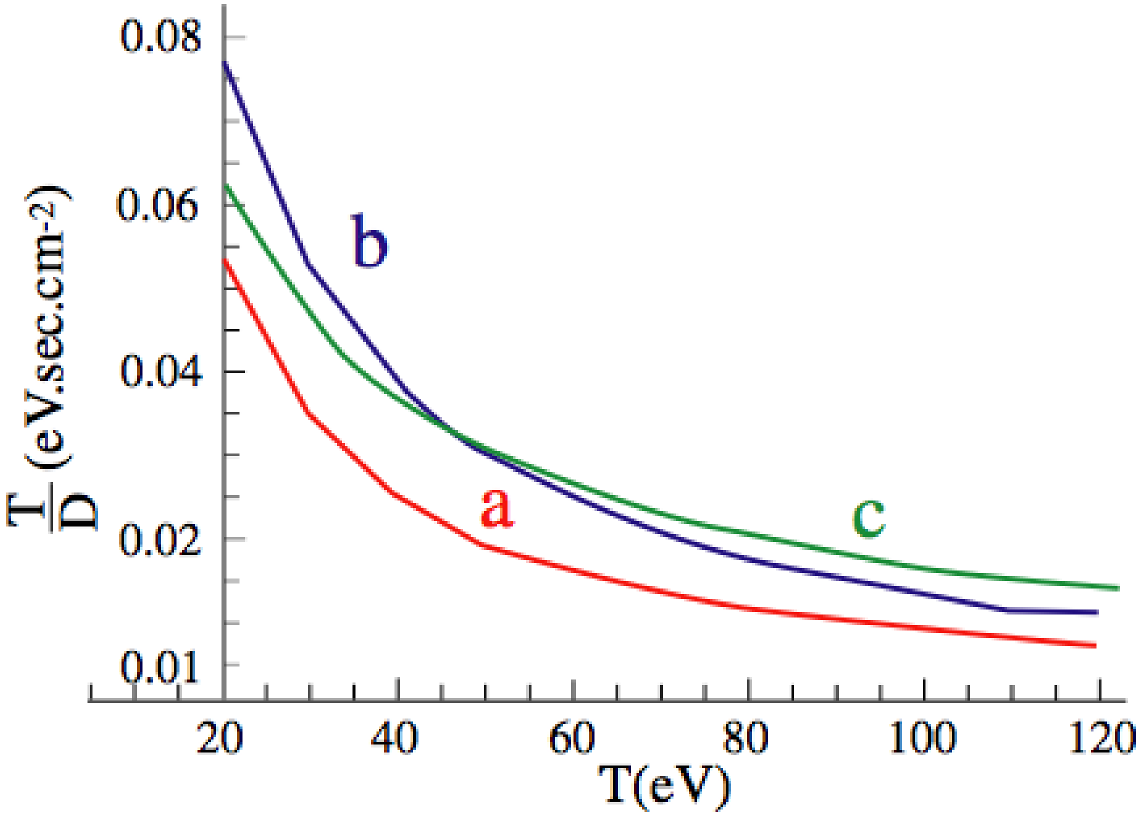
Expressions (12) and (13) are quantitatively equivalent, thus providing a theoretical template against which to evaluate LVISD deduced from OCP diffusion coefficients. The second advocated route implements the small ε_p approximation for that B = 0 self-diffusion coefficient given by Sjögren et al., under the form

$$D_2 = \frac{104\,Z^2 + 111\sqrt{2}\,Z + 59}{32\,Z^2 + 75\sqrt{2}Z + 50} D_2^{(1)} \qquad (14)$$

with

$$D_2^{(1)} = a^2\,\omega_p\left(\frac{\pi}{3}\right)^{1/2}\left((Z\sqrt{2}+1)\Gamma^{5/2}\ln\frac{k_{max}}{k_{min}}\right)^{-1} \qquad (14a)$$

For Z = 1, Eq. (14) gives D_2 = 1,70 $D_2^{(1)}$, with $D_2^{(1)}$ evaluated through a one Sonine polynomial approximation.

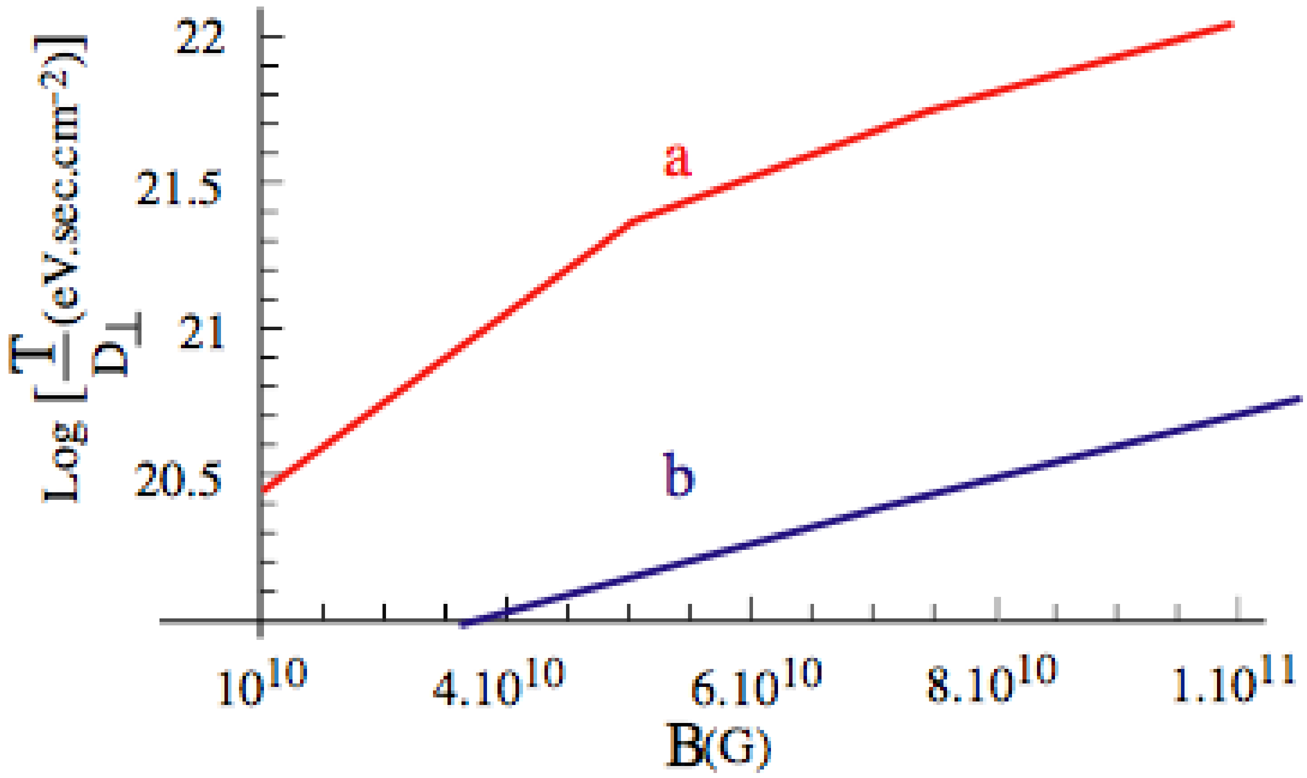


Low velocity proton stopping in dense plasma

($n=10^{21}$ e-cm^{-3}) in terms of target temperature

a) B = 0, LVISD (Eqs. (12,13), b) B=0, LVISD (Eq. (14))

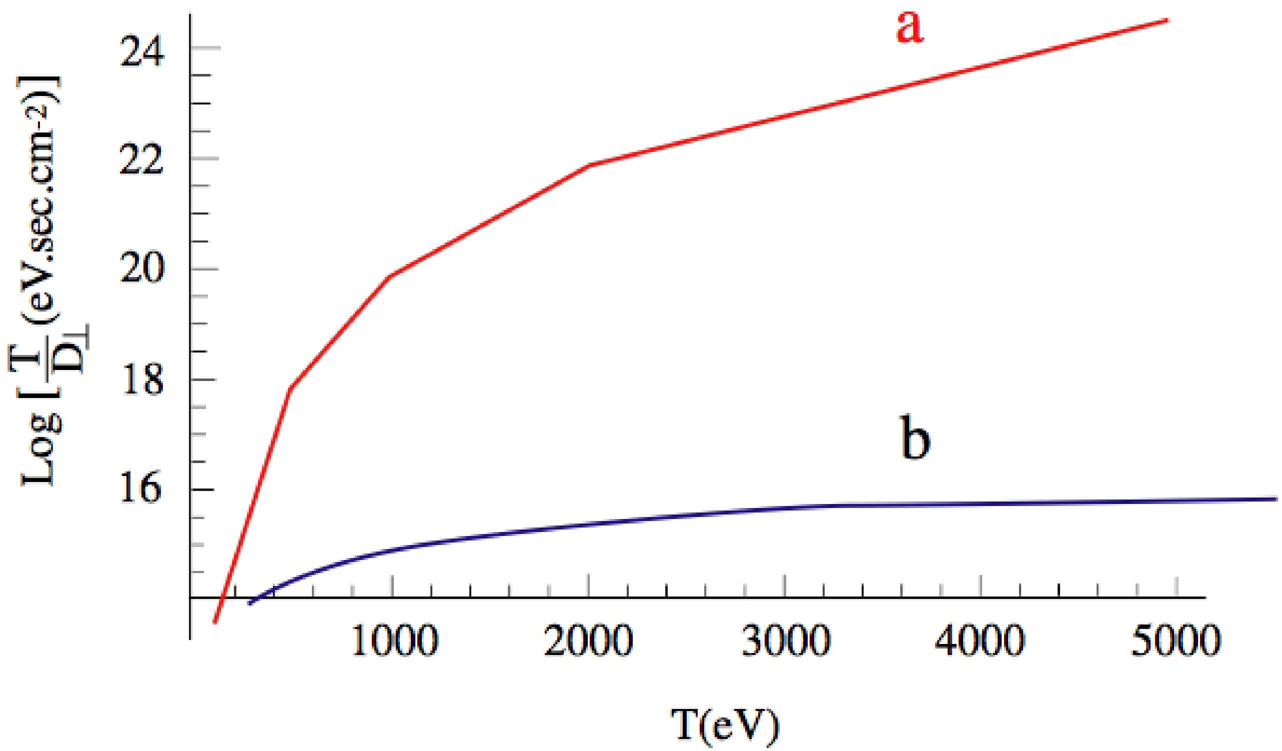
and c) parallel LVISD, Eq. (2) with Eq. (9a)



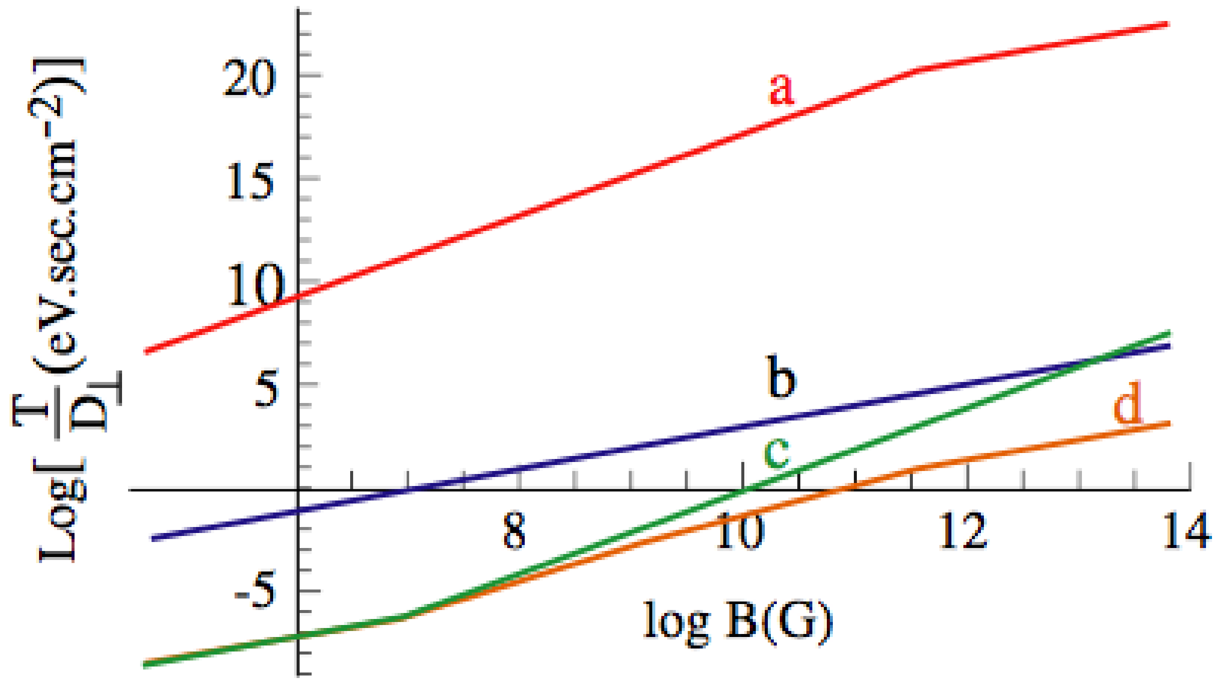
Proton transverse LVISD in a dense target ($n=10^{21}$ e-cm^{-3} T= 1 keV) In terms of B(G) in the Bohmlike ($D_\perp \sim B^{-1}$) approximation

a) Target electron contribution to stopping

b) Target ion contribution to stopping



Proton transverse LVISD in a dense plasma
($n = 10^{21}$ e-cm^{-3}, 100≤T(eV)≤5000 and B = 10^{10} G) in terms of T(eV)
a) electron stopping b) ion stopping



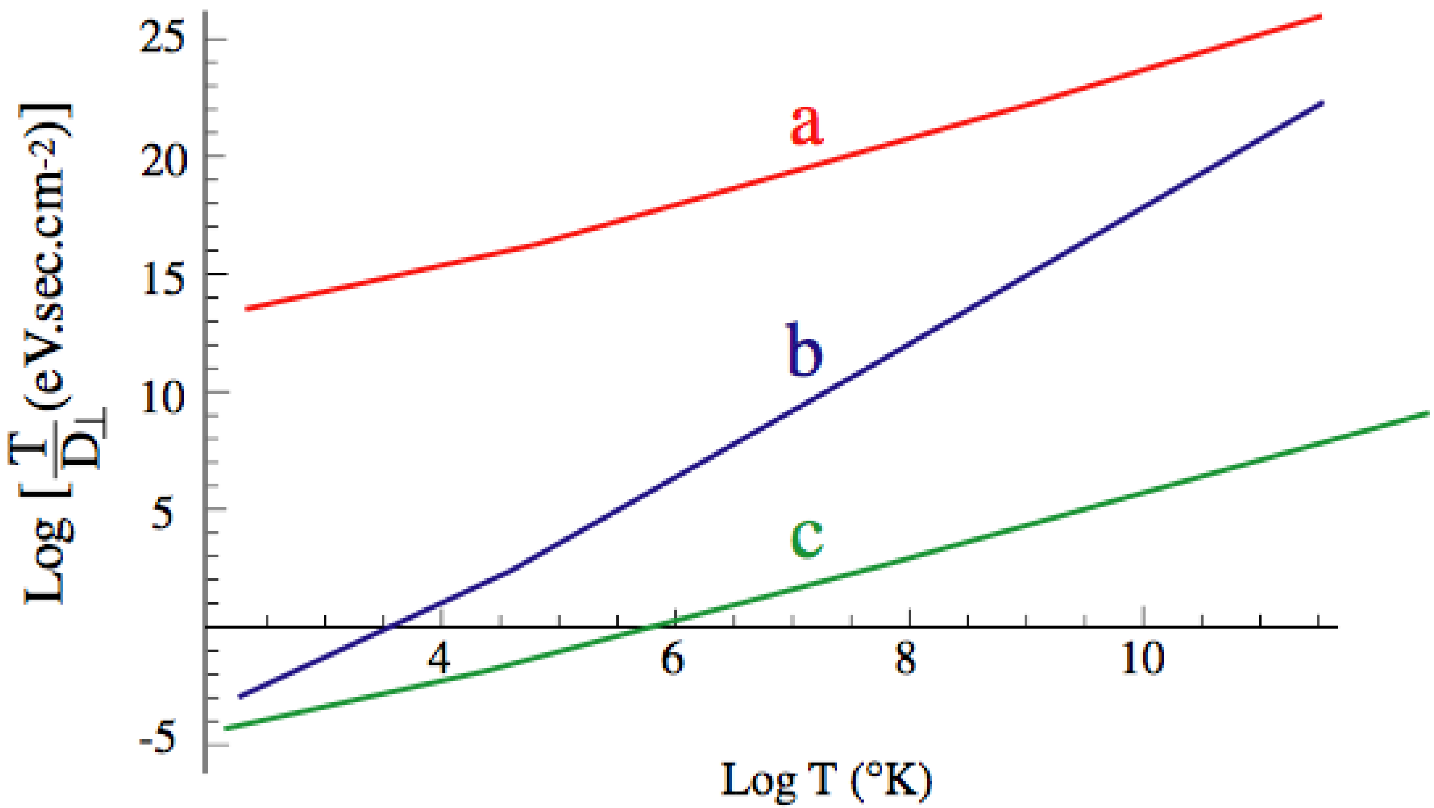
(n = 3.5x10^7 e-cm^{-3}, T = 100° K) in term of $10^2 \le B(G) \le 10^6$

a) Target electron slowing down ($D_\perp \sim B^{-2}$)

b) Target ion slowing down ($D_\perp \sim B^{-2}$)

c) Target electron slowing down ($D_\perp \sim B^{-1}$)

d) Target ion slowing down ($D_\perp \sim B^{-1}$)



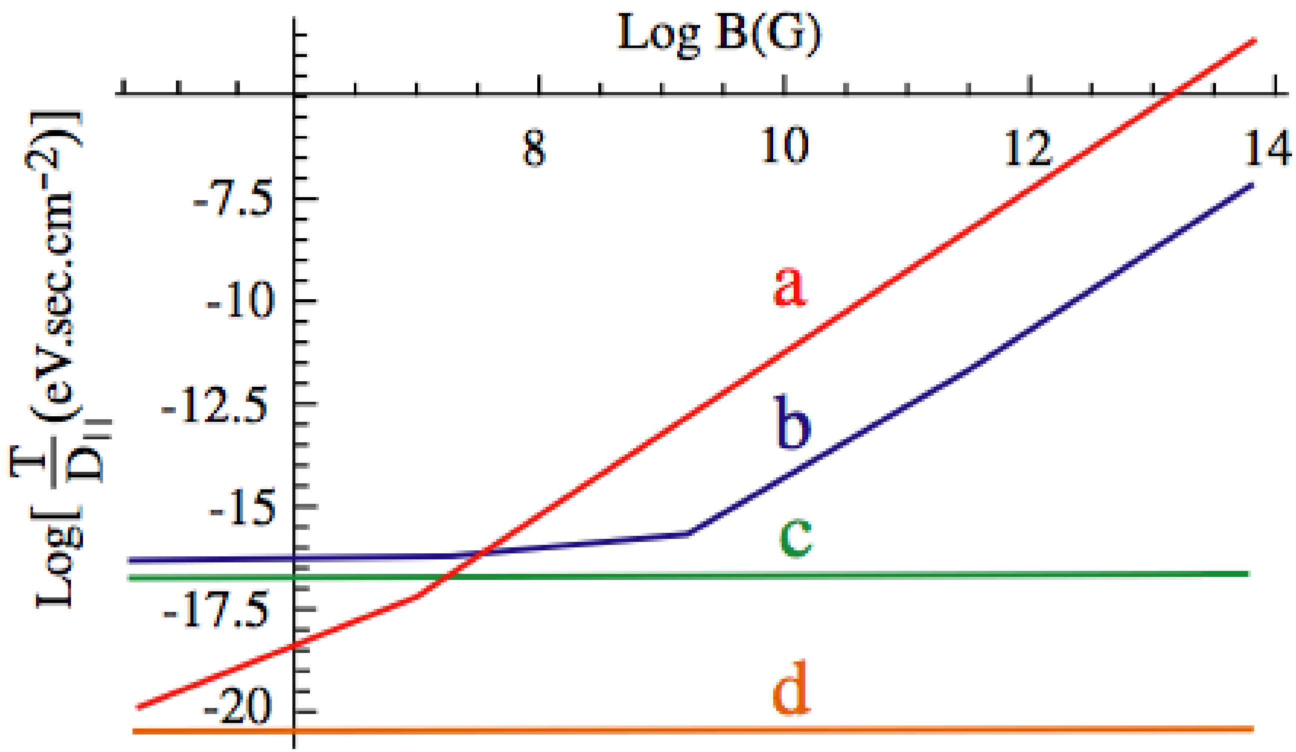
Proton transverse LVISD in a cold plasma

(n = 3.5×10^{7} e-cm^{-3}, $10 \le T(°K) \le 10^{5}$ and B= 10^{4} G) in term of T(° K)

a) electron stopping ($D_{\perp}\sim B^{-2}$)

b) electron stopping ($D_{\perp}\sim B^{-1}$)

c) Ion stopping



Proton parallel LVISD in a cold plasma

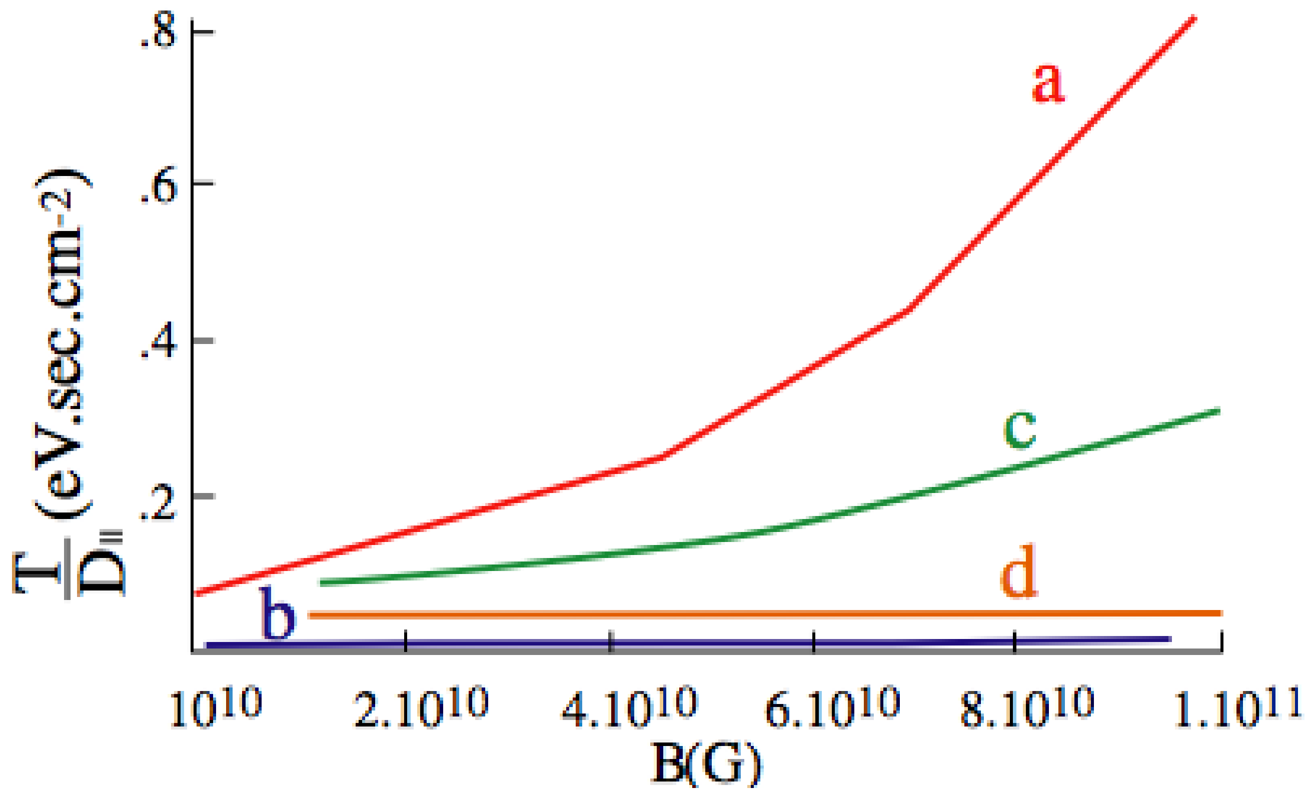
(n = 3.5×10^7 e-cm^{-3}, T = 100° K) in terms of magnetic intensity $10^2 \leq B(G) \leq 10^6$

a) Target electron slowing down for B≠0

b) Target ion slowing down for B≠0

c) Target ion slowing down with B-independent D

d) Target electron slowing down with B-independent D



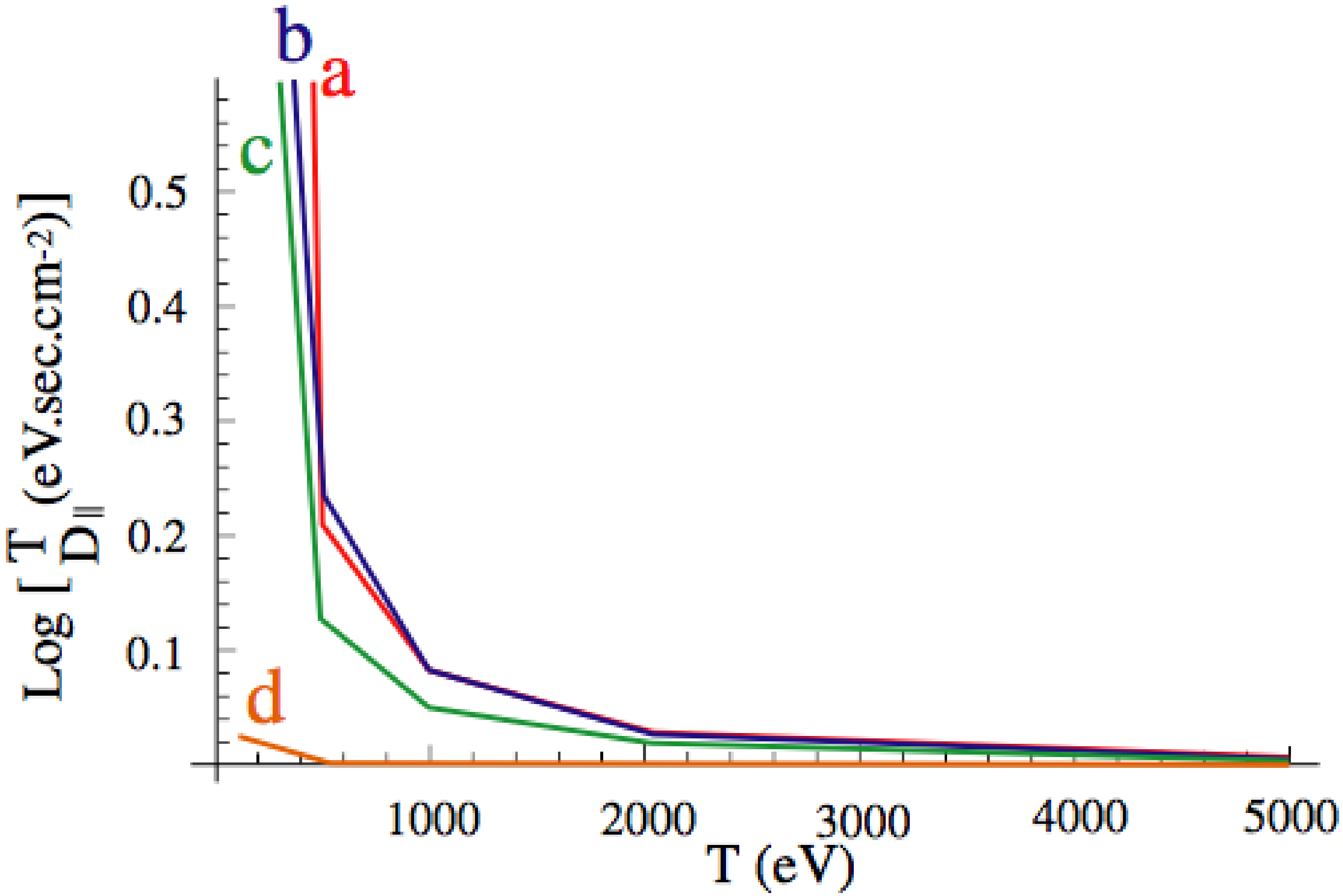
Proton parallel LVISD in a dense target ($n=10^{21}$ e-cm^{-3}, T=1 keV) in terms of B(G)

a) Kinetic electron contribution to stopping

b) Classical electron contribution to stopping

c) Kinetic ion contribution to stopping

d) Classical ion contribution to stopping



Proton parallel LVISD in a dense plasma

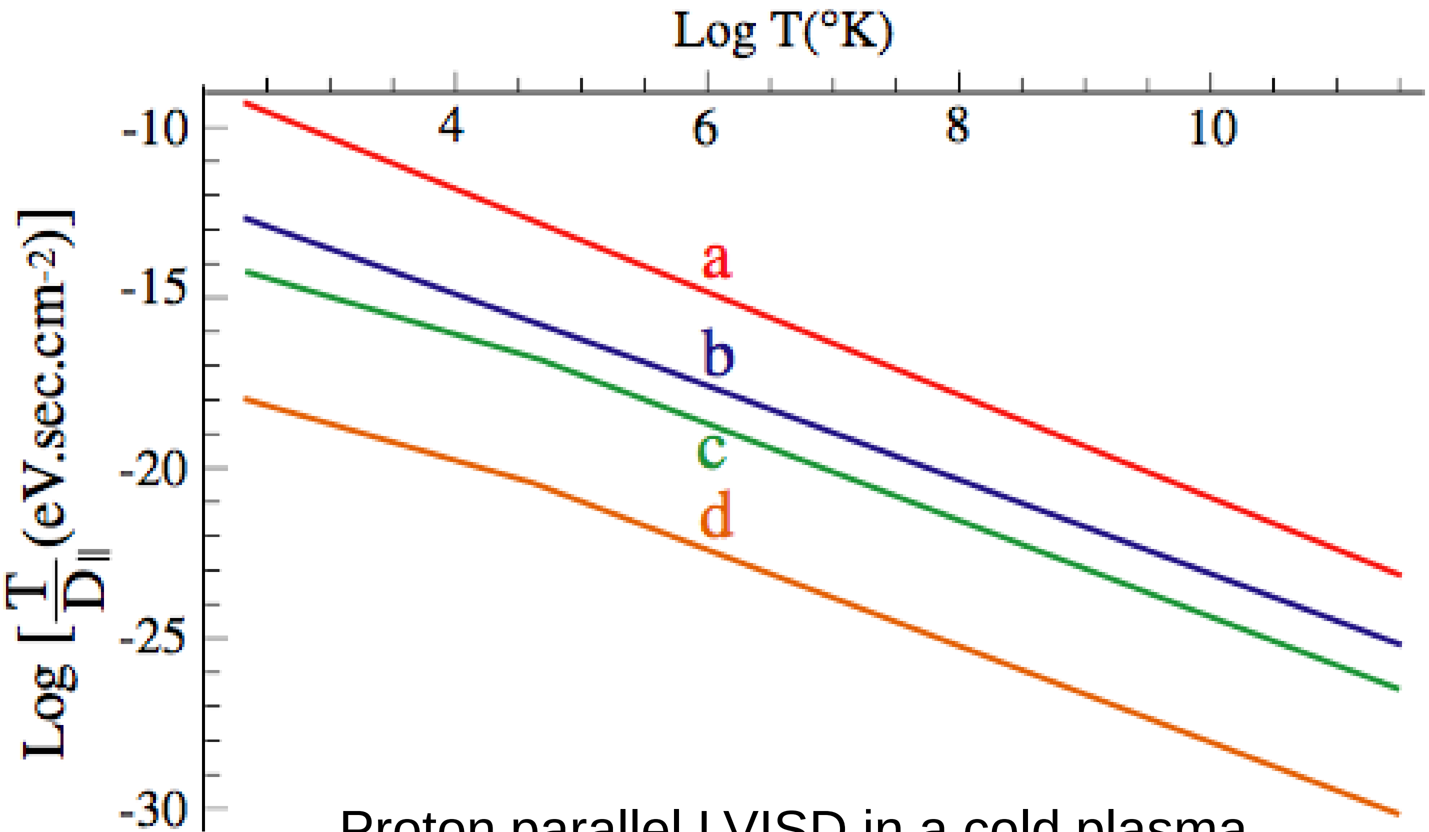
($n = 10^{21}$ e-cm^{-3}, $100 \leq T(eV) \leq 5000$ and $B = 10^{10}$ G) in terms of T(eV)

a) electron stopping ($B \neq 0$)

b) ion stopping ($B \neq 0$)

c) ion stopping ($B = 0$)

d) electron stopping ($B = 0$)



Proton parallel LVISD in a cold plasma
($n = 3.5\times10^{7}$ cm^{-3}, $B=10^{4}$ G), $10 \le T(°\ K) \le 10^{5}$)

a) target ion slowing down ($B\neq0$)

b) target ion slowing down ($B=0$)

c) target electron slowing down ($B\neq0$)

d) target electron slowing down ($B=0$)

(b) and (c) stand in a Log (43) ratio

T- dependence

The temperature behavior is much more intriguing, as respectively displayed for transverse and parallel LVISD in the highly strongly magnetized and dense target already considered for fast ignition in ICF. One then witnesses a monotonous increase for transverse stopping contrasted to a monotonous decay for the parallel counterpart.

SUMMARIES

-As a summary, we implemented very simple LVISD expression to the a priori very involved ion beam-arbitraly magnetized plasma interaction.

-We used transverse and parallel diffusion coefficients in suitably framed magnetized one-component-plasma (OCP) with target electrons building up the corresponding neutralizing background.

-Thus, we reached analytic LVISD transverse and parallel expressions advocating contrasting temperature behaviors.

-These quantities are of obvious significance in asserting the confinement capabilities of a very large scope of dense and strongly magnetized plasmas ranging from ultracold ones to those featuring the highest B values one can produce in the laboratory or observe in astrophysics.